GEOMETRICALLY NON-LINEAR FREE AND FORCED VIBRATION OF FULLY CLAMPED LAMINATED COMPOSITE SKEW PLATES

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ABSTRACT

In this paper, a theoretical model based on Hamilton’s principle and spectral analysis is used to obtain the geometrically non linear free and steady state forced response of a laminated skew plate at large vibration amplitudes. Such a structure is analyzed regarding the influence of different parameters: the intensity of the excitation force, the ply properties, the plate aspect ratio and skew angle. The solution of the amplitude equation is obtained in each case using the explicit analytical approach previously developed. The results showed, as may be expected due to the membrane forces induced by the large vibration amplitudes, a non linearity of the hardening type with a shift to the right of the bent frequency response function, in the neighborhood of the fundamental mode. The effects of the various parameters mentioned above have been examined and the comparison between the results obtained and those available in previous studies showed a good agreement.

1 INTRODUCTION

Field like Aerospatiale, mechanical and civil engineering are commonly used the thin laminated composite skew plates on their applications. Generally, such structures are supported a forces, and vibrating in high amplitude, inducing a new behavior in the lamina constituted the composite material. Analytical methods are interesting to understand the influence of different parameters on the response of the structure, and complete the numerical methods as a basic reference tool. A lot of studies are concerned by analytical and numerical method. Kadiri and Benamar [1-3] has developed a semi analytical method based on Hamilton's principle and spectral analysis, for determination of the geometrically non-linear free and forced response of thin straight structures. Two models for non-linear vibration of beam and plate have been proposed. These two models were based on the linearization of the nonlinear algebraic equations, written in the modal basis, in the neighbourhood of each resonance. The first formulation leads to explicit analytical expressions for higher mode contribution coefficients to the ith non linear mode shape of the structure considered, as functions of the amplitude of vibration, the mass, rigidity, and non linearity tensors. This first model was shown to be applicable to finit amplitude of vibration, up to 0.8 times the beam thickness, and 0.5 times the plate thickness. The second formulation was leaded to similar results for higher amplitudes of vibration, up to 2.3 time the beam thickness, and once the plate thickness via solution of reduced linear systems. Das and al analysed [4] the static behaviour of thin isotropic skew plates under uniformly distributed load with the geometric nonlinearity using a variational method based on total potential energy. Duan and Mahendran [5] analysed the large deflection behavior of skew plate with various skew angles, length to width ratios, thicknesses and supported edges under uniformly distributed and concentrated loads using a new hybrid/mixed shell element. Also, published works devoted to the forced vibration of composite plates was found in literature. Han and Petyt [6] investigated the forced vibration of symmetrically laminated plates using the hierarchical finit element method (HFEM). Nguyen-Van and al [7] presented an improved finite element computational model using a flat four-node element with smoothed strains for geometrically nonlinear analysis of composite plate/shell structures. The Von-Karman's large deflection theory and the total Lagrangian approach are employed in the formulation of the element to describe small strain geometric nonlinearity with large deformations using the first-order shear deformation theory (FSDT). Harras and Benamar [8, 9] investigated theoretical and experimental of the non-linear behavior of various fully clamped rectangular composite panels at large vibration amplitudes. Ribeiro and Petyt [10] has applying the principle of virtual work and the HFEM for studying the steady state, geometrically non-linear, forced vibration of isotropic and composite laminated rectangular plates under harmonic external excitation.

This work presents an explicit analytical model for the steady state, geometrically non linear, periodic forced vibration of fully clamped thin skew composite plates, under harmonic external excitation. The theoretical model developed in [1-3] was adapted here. Comparison was made between the iterative method and the approximate explicit method. The frequency response curves have been obtained at the plate centre, for various levels of loading, various skew angles and various aspect ratios. It appeared that the method works well, since excellent agreement was found between the result of the present model and those published in the literature.

2 EXPLICIT ANALYTICAL FORMULATION FOR THE GEOMETRICALLY NONLINEAR LAMINATED SKEW PLATE EXCITED HARMONICALLY BY CONCENTRATED OR DISTRIBUTED FORCES

Consider the skew plate with a skew angle θ shown in Figure 1. For the large vibration amplitudes formulation developed here, it is assumed that the material of the plate is elastic, isotropic and homogeneous. The thickness of the plate is considered to be sufficiently small so as to avoid the effects of shear deformation. The skew plate has the following characteristics: a, b, S: length, width and area of the plate; x-y: plate co-ordinates in the length and the width directions; ξ-η, H: Skew plate co-ordinates and plate thickness; E, ν: Young's modulus and Poisson's ratio; D, ρ: plate bending stiffness and mass per unit volume.

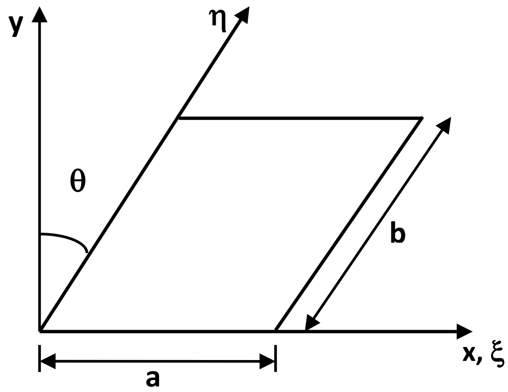

Figure 1 Skew plate in x-y and ξ-η co-ordinate system

For the classical plate laminated theory, the strain-displacement relationship for large deflections are given by:

$$\{\varepsilon\} = \{\varepsilon^0\} + z\,\{k\} + \{\lambda^0\}. \tag{1}$$

In which $\{\varepsilon^0\}$, $\{k\}$ and $\{\lambda^0\}$ are given by:

$$\{\varepsilon^0\} = \begin{bmatrix} \dfrac{\partial U}{\partial x} \\ \dfrac{\partial V}{\partial y} \\ \dfrac{\partial U}{\partial y} + \dfrac{\partial V}{\partial x} \end{bmatrix}; \{k\} = \begin{bmatrix} k_x \\ k_y \\ k_{xy} \end{bmatrix} = \begin{bmatrix} \dfrac{-\partial^2 W}{\partial x^2} \\ \dfrac{-\partial^2 W}{\partial Y^2} \\ \dfrac{-2\ \partial^2 W}{\partial xy} \end{bmatrix}; \{\lambda^0\} = \begin{bmatrix} \lambda_x^0 \\ \lambda_y^0 \\ \lambda_{xy}^0 \end{bmatrix} = \begin{bmatrix} \dfrac{1}{2}\left(\dfrac{\partial W}{\partial x}\right)^2 \\ \dfrac{1}{2}\left(\dfrac{\partial W}{\partial y}\right)^2 \\ \dfrac{\partial W}{\partial x}\dfrac{\partial W}{\partial y} \end{bmatrix}. \tag{2}$$

U, V and W are displacements of the plate mid-plane, in the x, y and z directions respectively. For the laminated plate having n layers, the stress in the Kth layer can be expressed in terms of the laminated middle surface strains and curvatures as:

$$\{\sigma_k\} = [\bar{Q}]_k\{\varepsilon\}. \tag{3}$$

In which $\{\sigma\}_k^T = [\sigma_x \sigma_y \sigma_{xy}]$ and terms of the matrix $[\bar{Q}]$ can be obtained by the relationships given in reference [11]. The in-plane forces and bending moments in a plate are given by:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \{\varepsilon^0\} + \{\lambda^0\} \\ \{k\} \end{bmatrix}. \tag{4}$$

A, B and D are the symmetric matrices given by the following Equation 5. $[B] = 0$ for symmetrically laminated plates [12].

$$\left(A_{ij}, B_{ij}, D_{ij}\right) = \int_{-H/2}^{H/2} Q_{ij}^{(k)} (1, z, z^2)\, dz. \tag{5}$$

Here the $Q_{ij}^{(k)}$ are the reduced stiffness coefficients of the kth layer in the plate co-ordinates. The transverse displacement function W may be written as in reference [10] in the form of a double series:

$$W = \{A_k\}^T \{W\} \sin k\omega t. \tag{6}$$

Where $\{A_k\}^T = \{a_1^k, a_2^k, \dots, a_n^k\}$ is the matrix of coefficients corresponding to the kth harmonic, $\{W\}^T = \{w_1, w_2, \dots, w_n\}$ is the basic spatial functions matrix, k is the number of harmonics taken in to account, and the usual summation convention on the repeated index k is used. As in reference [13], only the term corresponding to k=1 has been taken into account, which has led to the displacement function series reduced, to only one harmonic: i.e.,

$$W = a_i\, w_i(x, y) \sin\omega t. \tag{7}$$

Here the usual summation convention for the repeated indexes i is used. i is summed over the range 1 to n, with n being the number of basic functions considered. The expression for the bending strain energy V_b, axial strain energy V_a and kinetic energy T are given in reference (Harras 2001) in the rectangular co-ordinate (x,y). The skew co-ordinates are related to the rectangular co-ordinate (ξ,η) by: $\xi = x - y \tan\theta$; $\eta = y/\cos\theta$. So, the strain energy due to bending V_b, axial strain energy V_a and kinetic energy T are given in the ξ-η co-ordinate system. In the above expressions, the assumption of neglecting the in plane displacements U and V in the energy expressions has been made as for the fully clamped rectangular isotropic plates analysis considered in reference [13]. Discretization of the strain and kinetic energy expressions can be carried out leading to:

$$V_b = \frac{1}{2}\sin^2(\omega t) a_i a_j k_{ij}\, ; V_a = \frac{1}{2}\sin^4(\omega t) a_i a_j a_k a_l b_{ijkl}\, ; T = \frac{1}{2}\omega^2 \cos^2(\omega t) a_i a_j m_{ij}. \tag{8}$$

In which m_{ij}, k_{ij} and b_{ijkl} are the mass tensor, the rigidity tensor and the geometrical non-linearity tensor respectively. Non-dimensional formulation of the non-linear vibration problem has been carried out as follows.

$$w_i(\xi, \eta) = H w_i^*\left(\frac{\xi}{a}, \frac{\eta}{b}\right) = H w_i^*(\xi^*, \eta^*). \tag{9}$$

Where ξ^* and η^* are non-dimensional co-ordinates $\xi^* = \frac{\xi}{a}$ and $\eta^* = \frac{\eta}{b}$ one then obtains:

$$k_{ij} = \frac{aH^5E}{b^3}k^*_{ij};\; b_{ijkl} = \frac{aH^5E}{b^3}b^*_{ijkl};\, m_{ij} = \rho H^3 abm^*_{ij}. \tag{10}$$

Where the non-dimensional tensors m^*_{ij}, k^*_{ij} and b^*_{ijkl} are given in terms of integrals of the non-dimensional basic function w_i^*, non-dimensionnal extensional and bending stiffness coefficient A^*_{ij} and D^*_{ij} , skew angle θ and aspect ratio α.

Upon neglecting energy dissipation, the equation of motion derived from Hamilton's principle is:

$$\delta\int_0^{2\pi}(V-T) = 0. \tag{11}$$

Where $V=V_a+V_b$. Insertion of Equations 8 into Equation 11, and derivation with respect to the unknown constants a_i, leads to the following set of non-linear algebraic equations:

$$2a_i k^*_{ir} + 3a_i a_j a_k b^*_{ijkr} - 2\omega^* a_i m^*_{ir} = 0. \tag{12}$$

Where r=1, …, n. These have to be solved numerically. To complete the formulation, the procedure developed in [8] is adopted to obtain the first non-linear mode. As no dissipation is considered here, a supplementary equation can be obtained by applying the principle of conservation of energy, leads to the equation:

$$\omega^{*2} = \frac{a_i a_j k^*_{ij} + (3/2)a_i a_j a_k a_l b^*_{ijkl}}{a_i a_j m^*_{ij}}. \tag{13}$$

This expression for ω^{*2} is substituted into Equation 12 to obtain a system of n non-linear algebraic equations leading to the contribution coefficients a_i, i=1 to n. ω and ω* are the non-linear frequency and non-dimensional non-linear frequency parameters related by:

$$\omega^2 = \frac{D}{\rho b^4\cos^4\theta}\omega^{*2}. \tag{14}$$

To obtain the first non-linear mode shape of the skew plate considered, the contribution of the first basic function is first fixed and the other basic functions contributions are calculated via the numerical solutions of the remaining (n-1) non-linear algebraic equations.

In this section, a fully clamped laminate skew plate excited by a concentrated harmonic force Fc applied at the point (ξ_0,η_0); or by a distributed harmonic uniform force F^d, distributed over the surface of the plate S are considered. F^c and F^d may be written using the Dirac function δ as:

$$F^c(\xi,\eta,t) = F^c\delta(\xi-\xi_0)\delta(\eta-\eta_0)\sin\omega t. \tag{15}$$

$$F^d(\xi,\eta,t) = F^d\sin\omega t \quad \text{if } (\xi,\eta)\in S. \tag{16}$$

$$F^d(\xi,\eta,t) = 0 \quad \text{if } (\xi,\eta)\notin S. \tag{17}$$

The corresponding generalized forces $F_i^c(t)$ and $F_i^d(t)$ in the beam function basis (BFB) are given by:

$$F_i^c(t) = F^c w_i(x_0, y_0) \sin\omega t = f_i^c \sin\omega t. \tag{18}$$

$$F_i^d(t) = F^d \sin\omega t \int_\Omega w_i(x, y)\, dxdy = f_i^d \sin\omega t. \tag{19}$$

The explicit analytical method has been successively applied in references [1-3] to non-linear free and forced vibrations, occurring at large displacements amplitudes, of rectangular plate. The purpose of this paper is to apply the explicit simple formulation to non-linear forced vibrations of laminated skew plate, then, make comparison of the new results with those found by the iterative method and with the previous ones available in the literature in order to determine exactly the limit of validity of this formulation. Analytical details are given in this section for the first non-linear mode shape of a forced fully clamped laminated skew plate. As it was noticed that the contribution a_1 remains significantly higher than a_2 to a_n, denoted in what follows as ε_2, $\varepsilon_3, \ldots$, ε_{18}, the main idea of the approach presented in references [1-3] was to simplify the non-linear expression $a_i a_j a_k b_{ijkr}$ in Equation 12, which involves summation for the repeated indices i, j , k over the range {1,2,…,n}, by neglecting both first and second order terms with respect to ε_i, i.e. terms of the type $a_1^2 \varepsilon_k b_{11kr}$ or of the type $a_1 \varepsilon_j \varepsilon_k b_{1jkr}$ so that the only remaining term is $a_1^3 b_{111r}^*$. The Equation 12 becomes:

$$(k_{ir}^* - \omega^{*2} m_{ir}^*)\varepsilon_i + \frac{3}{2} a_1^3 b_{111r}^* = f_r^*, r = 2,3, \ldots, 18. \tag{20}$$

Where f_i^{*c} and f_i^{*d} corresponding, respectively to the dimensionless generalized concentrated force F^c at point (ξ_0,η_0); and to the uniformly distributed force F^d over the surface Ω of the plate; The expressions obtained are:

$$f_i^{*c} = F^c \frac{b^3}{aEH^4} w_i^*(\xi_0^*, \eta_0^*). \tag{21}$$

$$f_i^{*d} = F^d \frac{b^4}{EH^4} \iint_\Omega w_i^*(\xi^*, \eta^*)\, d\xi^* d\eta^*. \tag{22}$$

As mentioned in reference [3], the above system permits one to obtain explicitly the basic function contributions ε_2, $\varepsilon_3, \ldots$, ε_{18} of the second and higher functions, corresponding to a given value of the assigned first basic function contribution a_1 if k_{ir}^*, for $i \neq r$, is assumed to be negligible compared to k_{rr}^*, and direct solution was as follows:

$$\varepsilon_r = \frac{f_r^* - \frac{3}{2} a_1^3 b_{111r}^*}{k_{rr}^* - \omega^{*2} m_{rr}^*}, r = 2,3, \ldots, 18. \tag{23}$$

It was shown in Reference [3] that the accurate explicit analytical solution corresponding to the non-linear free and forced vibration cases can be obtained only in the normal modes basis of the fully clamped plate considered (MFB). So, the problem of non linear forced laminated skew plate will also be formulated in this appropriate basis, using the notation of Reference [3].

The simplified theory presented in this subsection focuses on non-linear vibrations of plates using a multi-mode approach and taking into account the coupling between the higher vibration modes. The solution obtained in Equation (23) makes it possible to get directly the non-linear frequency response function in the neighbourhood of the first mode. This gives not only the displacement at the centre of the plate, as is usually the case, as a function of the non-linear frequency, but also the plate response spatial distribution on its whole area, for each level of excitation. The results obtained by this approach, are in good agreement with the experimental results in Reference [14].

3 RESULTS AND DISCUSSION

The aim of this section is to apply the theoretical model presented above to analyze the geometrical non-linear free and forced dynamic response of skew fully clamped symmetrically laminated plates in order to investigate the effect of non-linearity on the non-linear resonance frequencies and non-linear fundamental mode shape at large vibration amplitudes. Convergence studies are carried out, and the results are compared with those available from the literature through a few examples of laminated composite skew thin clamped plates with different fibre orientation and aspect ratio. The material properties, used in the present analysis are: Isotropic plate and composite laminated plate (graphite/epoxy) has five layers symmetrical angle-ply (45°,-45°, 45°,-45°, 45°); E_L =173 GN/m2; E_T= E_L/15.4 G_{LT}/E_T = 0.79; ν_{LT} = 0.3.

Where E, G and ν are Young's modulus, shear modulus and Poisson's ratio. Subscripts L and T represent the longitudinal and transverse directions respectively with respect to the fibres. All the layers are of equal thickness. Calculation was made by using 18 functions corresponding to three symmetric beam functions in the ξ direction and three symmetric beam functions in the η direction, and three anti-symmetric beam functions in the ξ direction and three anti-symmetric beam functions in the η direction. Table 1 shows the non linear results for a fully clamped isotropic square plate subjected to harmonic distributed force $f_1{}^{*d}$=104.45 (F^d=873.82N/m^2) obtained using a multimode approach. It can be seen a good convergence with results presented in reference [3].

W_{max}*	Reference [3]	Present result	Error %
+0.2	0.1475	0.1487	0.81
-0.2	1.4218	1.4220	0.01
+0.4	0.7661	0.7671	0.13
-0.4	1.2596	1.2602	0.05
+0.6	0.9285	0.9304	0.20
-0.6	1.2364	1.2377	0.11
+0.8	1.0476	1.0507	0.30
-0.8	1.2639	1.2665	0.21
+1	1.1588	1.1632	0.38
-1	1.3202	1.3240	0.29

Table 1. Forced vibration frequency ratio ω/ωl for a fully clamped square plate subject to harmonic distributed force $f_1{}^{*d}$=104.45 (F^d=873.82N/m^2).

The variation of non-dimensional nonlinear frequency ratio ω_{nl}/ω_l with respect to non dimensional maximum amplitude w_{max}/h is evaluated for different skew angle subjected to uniform harmonic load is shown in figure 2. The nonlinearity is reduced with increasing skew angle. It can be noticed multivalued regions corresponding to the jump phenomena occurring in non-linear vibration.

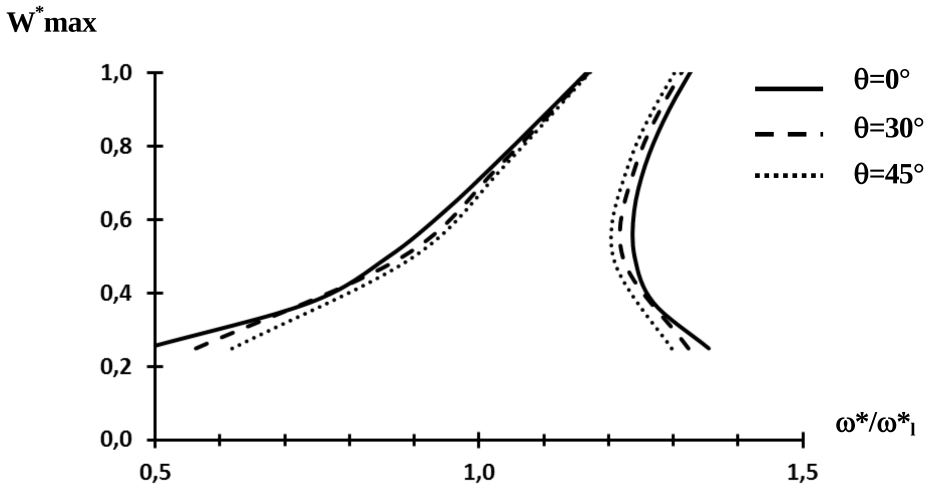

Figure 2. Comparison of the Forced response of a fully clamped isotropic square plate subjected to harmonic distributed force $f_1{}^{*d}$=104.45 (F^d=873.82N/m^2) for different skew angle.

A comparison between results obtained by the explicit model with those obtained using the single mode approach for fully clamped laminated composite plate excited by a harmonic distributed forces $f_1{}^{*d}$=10 (F^d=124.7N/m^2) was presented in Figure 3. It can be seen a reasonable estimate for the amplitude at the centre of the plate.

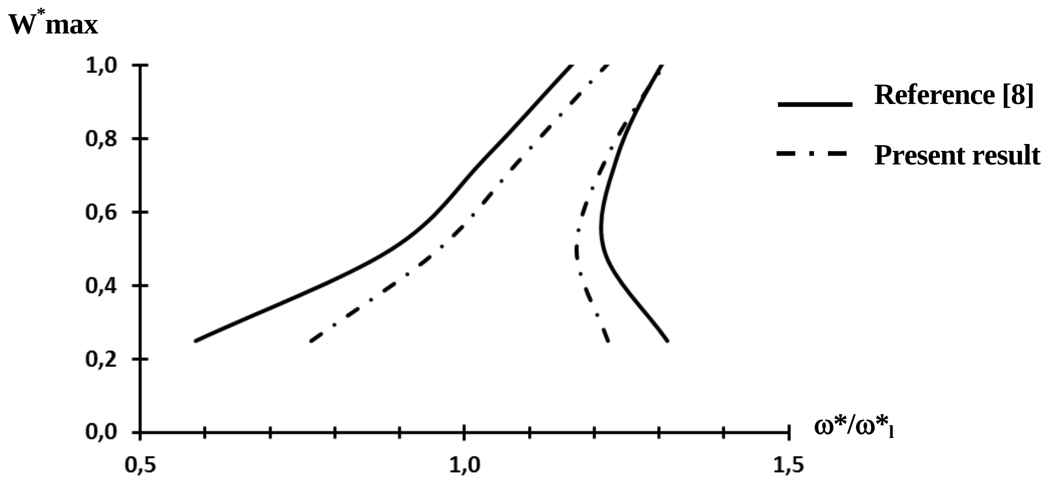

Figure 3. Comparison of the forced response of a fully clamped composite square plate subjected to harmonic distributed force $f_1{}^{*d}$=10 (F^d=124.7N/m^2) obtained with explicit method with reference [8].

In the case of fully clamped composite skew plate subjected to harmonic distributed force $f_1{}^{*d}$=10 (F^d=124.7N/m^2) with aspect ratio equal to 1, the effect of increasing skew plate on the nonlinearity was clearly exhibited in figure 4. The nonlinearity decreases with increases skew angle. For skew angle θ=45° it decreases about 10% compared with the rectangular case.

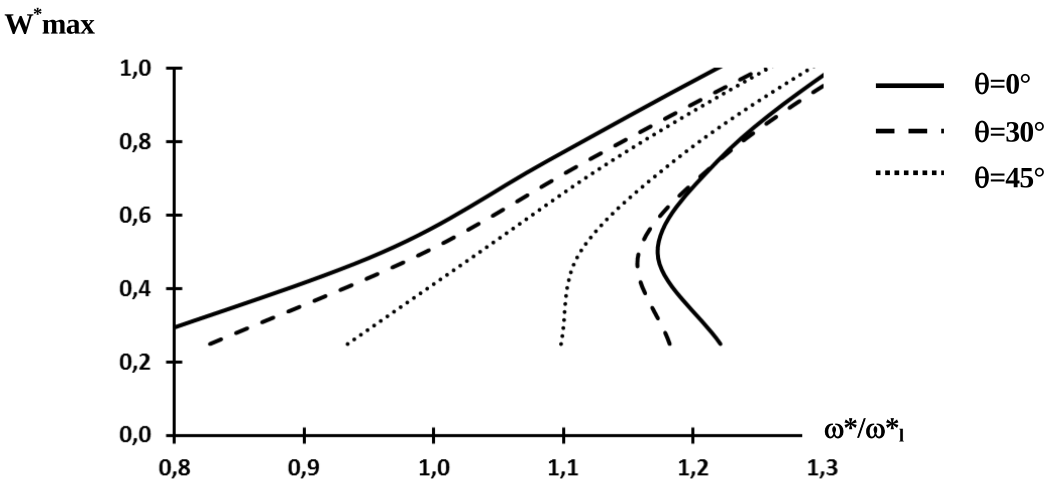

Figure 4. Comparison of the Forced response of a fully clamped composite plate subjected to harmonic distributed force $f_1{}^{*d}=10$ ($F^d=124.7N/m^2$) for different skew angle and $\alpha=1$.

The figure 5 shows the effect of the aspect ratio on the fully clamped composite skew plate. It can be seen that the increasing of aspect ratio; reduced the non-linearity of the plate.

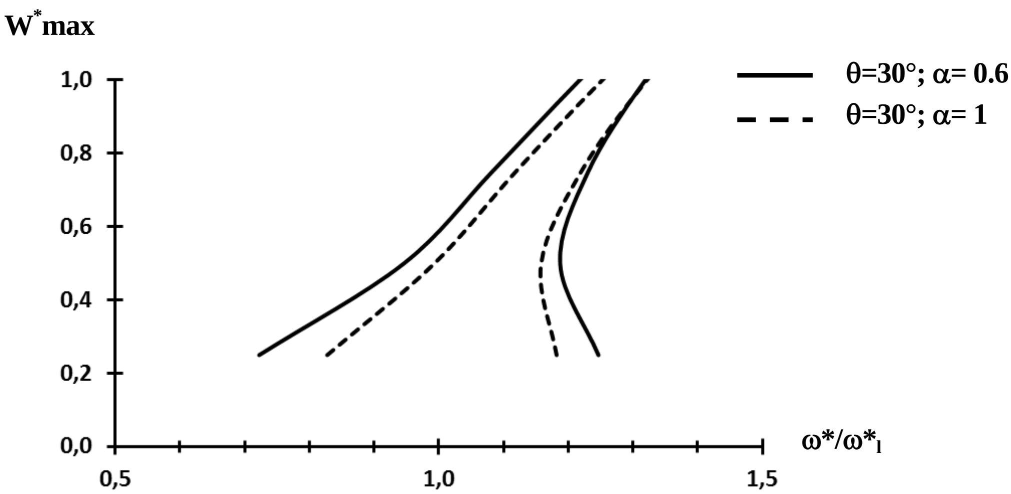

Figure 5. Comparison of the forced response of a fully clamped composite skew plate subjected to harmonic distributed force $f_1{}^{*d}=10$ ($F^d=124.7N/m^2$) for different aspect ratio α and $\theta=30°$.

4 CONCLUSION

A model using a semi analytical approach based on lagrange's equations, and the harmonic balance method are successively applied for geometrical non-linear, steady state, periodic forced vibration of composite laminated skew plates. Good results were found using a single and multimode approach to determine the amplitude frequency dependence in the centre of the plate by varying skew angle and aspect ratio. It can be seen that the skew angle reduce the effect of the nonlinearity, also the increasing aspect ratio decrease the nonlinearity. Good agreement between the present results and those found in literature has been achieved.

REFERENCES

[1] M. EL Kadiri, R. Benamar, and R. G. White. Improvement of the semi-analytical method, based on Hamilton's principle and spectral analysis, for determination of the

geometrically non-linear free response of thin straight structures. Part II: Application to the first and second non-linear mode shapes of fully clamped rectangular plates. *Journal of Sound and Vibration*. 257: 16-62, 2002.

[2] M. EL Kadiri, and R. Benamar. Improvement of the semi-Analytical Method, for Determining the Geometrically non-linear Response of Thin Straight Structures. Part I: Application to Clamped-Clamped and Simply Supported Clamped Beams. *Journal of Sound and vibration*. 249: 263-305, 2002.

[3] M. El Kadiri, and R. Benamar. Improvement of the semi-analytical method, based on Hamilton's principle and spectral analysis, for determination of the geometrically non-linear response of thin straight structures. Part III: steady state periodic forced response of rectangular plates. *Journal of Sound and Vibration*. 264: 1-35, 2003.

[4] D. Das, P. Sahoo, and K. Saha. A variational analysis for large deflection of skew plates under uniformly distributed load through domain mapping technique. *International Journal of Engineering, Science and Technology*. 1(1): 16-32, 2009.

[5] M. Duan, and M. Mahendran. Large deflection analyses of skew plates using hybrid/mixed finite element method. *Computers and Structures*. 81:1415-1424, 2003.

[6] W. Han, and M. Petyt. Linear vibration analysis of laminated rectangular plates using the hierarchical finite element method-II: Forced vibration analysis. *Computers and Structures*. 61 (4):713-724, 1996.

[7] H. Nguyen-Van , N. Nguyen-Hoai, T. Chau-Dinh, and T. Nguyen-Thoi. Geometrically nonlinear analysis of composite plates and shells via a quadrilateral element with good coarse-mesh accuracy. *Composite Structures*. 112: 327-338, 2014.

[8] B. Harras. Theoretical and Experimental Investigation of the non-Linear Behaviour of various Fully Clamped Rectangular Composite Panels at large Vibration Amplitudes. *Ph.D. thesis, University Mohammed V-Agdal*. 2001.

[9] B. Harras, R. Benamar, and RG. White. Investigation of non-linear free vibrations of fully clamped symmetrically laminated carbon-fibre-reinforced PEEK (AS4/APC2) rectangular composite panels. *Composites sciences and technology*. 62: 719-727, 2002.

[10] P. Ribeiro, and M. Petyt. Geometrical non-linear, steady state, forced, periodic vibration of plates. Part I: Model and convergence studies. *Journal of Sound and Vibration*. 226 (5): 955-983, 1999.

[11] S. Timoshenko, S. Weinsowsky-Krieger, and RM. Jones. *Mechanics of Composite Materials, 51*. International Student Edition, McGraw-Hill Kogakusha, Ltd. Tokyo, 1975.

[12] P.K. Mallick. *Fibre-reinforced composites: Materials, Manufacturing and Design, 152*. New York: Marcel Dekker, Inc., second edition revised and expanded, 1993.

[13] R. Benamar, M.M.K. Bennouna, and R.G. White. The effects of large vibration amplitudes on the fundamental mode shape of thin elastic structures. Part II fully clamped rectangular isotropic plates. *Journal of Sound and Vibration*. 164: 295-316, 1993.

[14] R. Benamar, M.M.K. Bennouna, and R.G. White. The effects of large vibration amplitudes on the mode shapes and natural frequencies of thin elastic structures. Part III: fully clamped rectangular isotropic plates-measurment of the mode shape amplitude dependence and spatial distribution of harmonic distortion. *Journal of sound and vibration*. 175: 377-395, 1991.