A TWO STEPS DAMAGE LOCALIZATION METHOD BASED ON WAVELET PACKET DECOMPOSITION. APPLICATION TO MULTI-LAYER COMPOSITE STRUCTURES

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ABSTRACT

In this work, we propose a two steps method for localizing damages in multi-layer composite structures. The signals of the dynamic responses of the healthy and damaged structures are initially decomposed using the analysis into wavelet packages and then rebuilt. Then, energies of these last signals are used to define an indicator of variation of energy called Wavelet Packages based Energy Variation Index (WPEVI). The robustness of this damage localization index is tested in the case of composite beams with respectively two and three damages.

To ascertain the quality of the results, the sensitivity of the proposed method to structural damage is studied. In this respect, we define the threshold sensibility of a damage vibration indicator based on wavelet package decomposition of structural vibration responses before and after the occurrence of structural damage. Each of these structural vibration responses is decomposed to the j^{th} *order wavelet package sub-signals. The structure is subdivided into a certain number of finite elements. For a structure with one particular finite element perturbed to a certain rate, the damage indicator is then defined as the maximum of all energy variations of the wavelet package sub-signals of the structural vibration response before and after the occurrence of some structural damage. The indicator is called maximum energy variation (MEV). For the same mono-excitation, this indicator is then evaluated for all the elements of the discretization with the same perturbation rate. The values of this indicator are then represented on a graph in terms of the number of the finite element. Once we have determined the element whose damage indicator value is minimum, further trials are carried out in order to draw the curve representing the indicator in terms of the different rates of damage. From the graph, we determine the least detectable damage rate using the wavelet package decomposition of structural response. A mapping of the structure is carried out as to the least detectable damaged element of the structure. With this tool in hand, we may ascertain or not each of the damage localization results.*

1 INTRODUCTION

The aim of this work is to develop a damage detection-localization procedure as well as a global damage threshold indicator, both based on the wavelet packet decomposition.

The projected damage detection procedure is to use temporal response function data. The method is in particular based on sub-signals of a certain level in the decomposition process of a signal in wavelet packets. The property of wavelet packets decomposition for denoizing signals is certainly of great help particularly in the case of laminated composite structures.

Another part of the undertaken work consists in establishing a global threshold damage indicator. Many research works have dealt with damage detection in composite structures but few of these have been concerned with the minimum of the magnitude of the damage that can be detected. Yam and al [1] use the wavelet packet analysis for the detection of damage in composite laminate plates. In their work, they define a damage indicator named "Maximum Energy Variation" (MEV). To establish this, they consider a structure with one particular finite element perturbed to a certain rate. Then, they calculate successively the energy of each sub-signal in the wavelet packets decomposition of the dynamical response of the damaged structure as well as that of the corresponding sub-signal corresponding of the healthy structure. The difference of these respective energies is then calculated and divided by the total energy of the sub-signals of the considered level of wavelet decomposition. The maximum of these normalized variations constitutes the so-called MEV damage indicator.

Our contribution consists in finding a global threshold damage indicator rather than a particular one for a defined position of the damaged element in the structure as did Yam and al. This global threshold damage indicator is useful to help ascertain whether results of the localization process are reliable or not.

In our work, we are interested in using this damage indicator in the case of a layered beam structure. For the purpose of analysis, a finite element model of this structure is built. For the same excitation and damage rate, a graph representing the indicator in terms of the perturbed element number is drawn to determine the element for which this has the lowest value. Different graphs of the MEV indicator in terms of damage rate may be drawn for different damaged element positions along the structure, and a damage detection threshold is established.

Before presenting the damage detection method and the global threshold damage indicator, first let us define the SI20 beam finite element.

2 FINITE ELEMENT SI20

The finite element model we use in this study is based on the theory of the zigzag movement of the first order. The finite element [2] is composed of three layers in symmetrical stackings sequences. Thus, the displacement at an arbitrary point of the beam can be expressed by a longitudinal displacement u_1 (x) along the beam axis and a transverse u_3 (x) along the z axis and a rotation γ_x (x) characterizing the rotation about the y axis (see FIG. 1).

Displacements are given by:

$$\left.\begin{aligned} u_1(x) &= u(x) + z\,\gamma_x(x) \\ u_3(x) &= w(x) \end{aligned}\right\} \tag{1}$$

where u is the longitudinal displacement at z = 0 and w is the deflection of the beam axis.

The rigidity matrix is obtained from the strain energy of the element:

$$U_e = \sum_{k=1}^{3} \frac{1}{2} \int_0^l \varepsilon^{(k)T}\, D^{(k)}\, \varepsilon^{(k)}\, dx \;=\; \sum_{k=1}^{3} \frac{1}{2} \int_0^l v_e{}^{(k)T} B^{(k)T}\, D^{(k)} B^{(k)} v_e{}^{(k)} dx \tag{2}$$

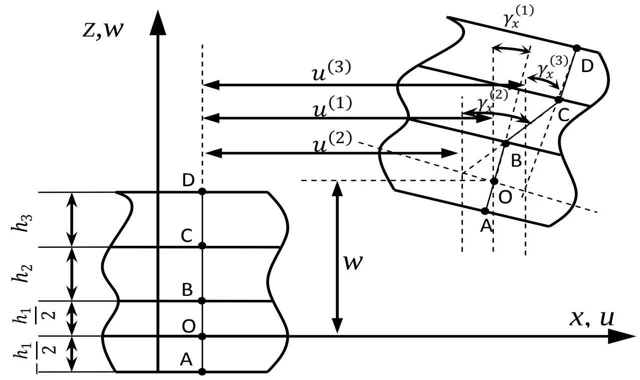


FIG. 1– Initial and deformed SI20 element

where k: layer number. k = 1, 2 et 3. l: length of the element. D: elasticity matrix of the element.

B: deformation matrix. $v_e = u^{(k)}, w^{(k)}, \gamma_x{}^{(k)}]^T$: nodal displacement vector.

Similarly, we may write the kinetic energy of the element in order to derive the mass matrix:

$$T_e = \sum_{k=1}^{3} \frac{1}{2} \int_0^l \dot{u}^{(k)T}\, R_0^{(k)} \dot{u}^{(k)} dx = \sum_{k=1}^{3} \frac{1}{2} \int_0^l \dot{v}_e^{(k)T}\, N^T R_0^{(k)} N \dot{v}_e dx \tag{3}$$

where : $\dot{v}_e$: the nodal velocity vector. N : the shape function matrix. R_0 : Matrix bulk densities.

$$R_0 = \begin{bmatrix} \rho_0 & 0 & \rho_1 \\ 0 & \rho_0 & 0 \\ \rho_1 & 0 & \rho_2 \end{bmatrix}$$

Generalized densities are given by:

$$\rho_0 = b\sum_{k=1}^{K} \rho_k \left[z_k - z_{k-1}\right];\ \ \rho_1 = \frac{1}{2} b\sum_{k=1}^{K} \rho_k \left[z_k^2 - z_{k-1}^2\right];\ \ \rho_2 = \frac{1}{3} b\sum_{k=1}^{K} \rho_k \left[z_k^3 - z_{k-1}^3\right] \tag{4}$$

Once the elementary matrices of rigidities and masses are obtained, they will be assembled to build the global stiffness matrices and weight of the individual system:

$$M = \sum_{i=1}^{n} M_i^{(e)} \quad \text{and} \quad K = \sum_{i=1}^{n} K_i^{(e)} \tag{5}$$

Having obtained the analytical model of the beam structure, in the case of a sine wave excitation, the equation of forced motion can be easily derived.

3 ENERGY CHANGE INDICATOR

Let $y(t)$ the signal of the dynamic response of a structure. This signal is decomposed by wavelets packets in a sum of subsignals $y_j^i(t)$, at the ith level, as follows:

$$y(t) = \sum_{i=1}^{2^j} y_j^i\,(t) \tag{6}$$

The energy U stored in a sub-signal is given by:

$$U_j^i = \int_{-\infty}^{+\infty} y_j^i(t)^2\,dt \tag{7}$$

Thus, the total energy U of the signal is defined as being the sum of the energies of these sub-signals

$$U = \sum_{i=1}^{2^j} U_j^i \tag{8}$$

We consider the responses of the two structures healthy indexed h and damaged indexed d. For each structure, we define respectively the vectors V_h and V_d whose components are each the ratio of the sub-signal energy to the total energy of the signal at the level selected of wavelet package decomposition.

$$V_h = \left\{C_{hj}^1, C_{hj}^2, C_{hj}^3, \cdots, C_{hj}^{2^j}\right\} = \left\{\frac{U_{hj}^1}{U_h}, \frac{U_{hj}^2}{U_h}, \frac{U_{hj}^3}{U_h}, \cdots \frac{U_{hj}^{2^j}}{U_h}\right\} \tag{9}$$

$$V_d = \left\{C_{dj}^1, C_{dj}^2, C_{dj}^3, \cdots, C_{dj}^{2^j}\right\} = \left\{\frac{U_{dj}^1}{U_d}, \frac{U_{dj}^2}{U_d}, \frac{U_{dj}^3}{U_d}, \cdots \frac{U_{dj}^{2^j}}{U_d}\right\} \tag{10}$$

From these last two vectors we define the relative energy variation vector as follows:

$$EV = \left\{\left(\frac{C_{hj}^1 - C_{dj}^1}{C_{hj}^1}\right), \left(\frac{C_{hj}^2 - C_{dj}^2}{C_{hj}^2}\right), \left(\frac{C_{hj}^3 - C_{dj}^3}{C_{hj}^3}\right), \cdots, \left(\frac{C_{hj}^{2^j} - C_{dj}^{2^j}}{C_{hj}^{2^j}}\right)\right\} \times 100\,\% \tag{11}$$

The MEV is defined as the maximum of the absolute relative energy variations (EV).

4 NUMERICAL SIMULATION TEST CASES

The composite beam structure represented in FIG.2 is composed of 3 layers glass/époxy, disposed in the following configuration $[\mathbf{0°/90°/0°}]$**. The** beam is subdivided into 60 finite elements SI20 [3].

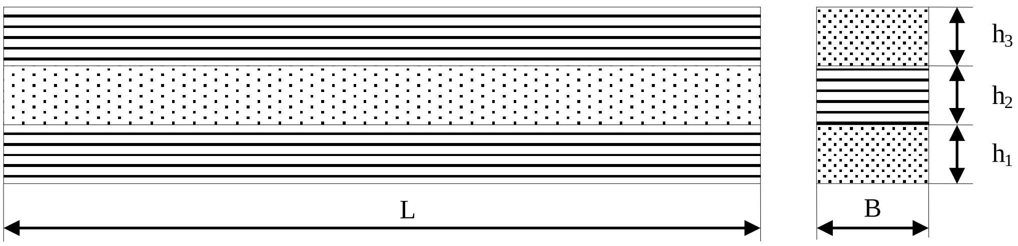


FIG. 2 – Stratified composite beam

The mechanical properties of the material are : E_1 = 47.518 GPa; E_2 = 4.588 GPa; G_{12} = 2.201 GPa; μ_{12} = 0.0419; μ_{21} = 0.434; ρ = 1850 kg/m^3. Its geometry is characterized by : L = 360 mm, $h_1 = h_2 = h_3$ = 4mm, B = 30 mm.

We consider two types of boundary conditions: a cantilever and a simply-simply supported beam. The damaged beam is simulated by reducing the longitudinal Young's modulus E_2 of the middle layer of the finite element.

For the first type of boundary conditions, the excitation is applied at its first node from the free end and for the second case of boundary conditions it is applied at the 25th node. The excitation force consists of three components and it is applied normally to the beam at the node previously indicated:

$$F_e(t) = A_0(\sin(\omega_1 t) + sin(\omega_2 t) + sin(\omega_3 t))$$

The measurement frequency band must contain as many eigenmodes as possible, and the exciting force must be chosen to excite the maximum number of these. The temporal responses signals of healthy and damaged structures are decomposed by «db4» wavelets packets to the 5th level.

5 HOW TO DETERMINE DAMAGE THRESHOLD OF A STRUCTURE

The strategy for determining damage threshold is highlighted through the stratified composite beam defined above in two cases of boundary conditions.

5.1 CANTILEVER BEAM CASE

To begin, we damage by 40% successively each element of the structure and we draw the MEV curve in terms the element number of the structure (FIG. 3).

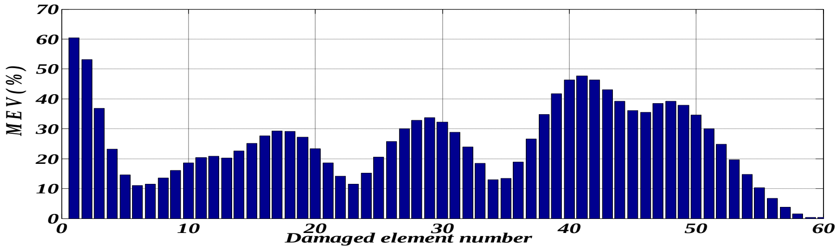


FIG. 3 – Cantilever beam : Histogram of MEV – 40% damaged beam element number.

The sensitivity of this indicator is variable according to the location of the defect. We choose on the FIG.3 element 6 which has the lowest value of MEV. We damage it at various rates and we seek in the waveband a combination of components of the exiting force which gives us the curve representative of MEV according to the rate of damage. This shape of the curve enables us to fix a priori a threshold of detectability of damages at the selected position (FIG. 4). The threshold is fixed just under the point of inflection of the curve at 60% and the smallest rate of detectable damage of element 6 is thus approximately 22%.

We obtain the same shape of the MEV-damage rate curve each time we change the position of the damaged element, the structure being always subjected to the same excitation as in the case of the damage of element 6, while varying the extent of the damage. This makes it possible to fix a threshold for each selected position of the damaged element. We represent above for some elements the graphs of MEV according to the damage rate of the element.

We maintain the same threshold of 60% for all the damaged elements and we take note of the smallest rate of detectable damage (FIG.5a). For element 12 for example, the smallest detectable rate is 20%; for element 18 it is 14%; for element 38 it is 12%.

5.2 SIMPLY-SIMPLY SUPPORTED BEAM CASE

Let us consider the case of the simply-simply supported beam. In the same manner as we proceeded in the preceding case, we start by damaging by 40% successively each element of the structure. The histogram representing the MEV according to the damaged element number is given by the FIG. 4.

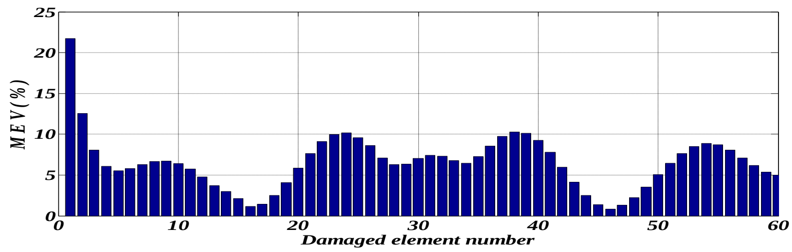
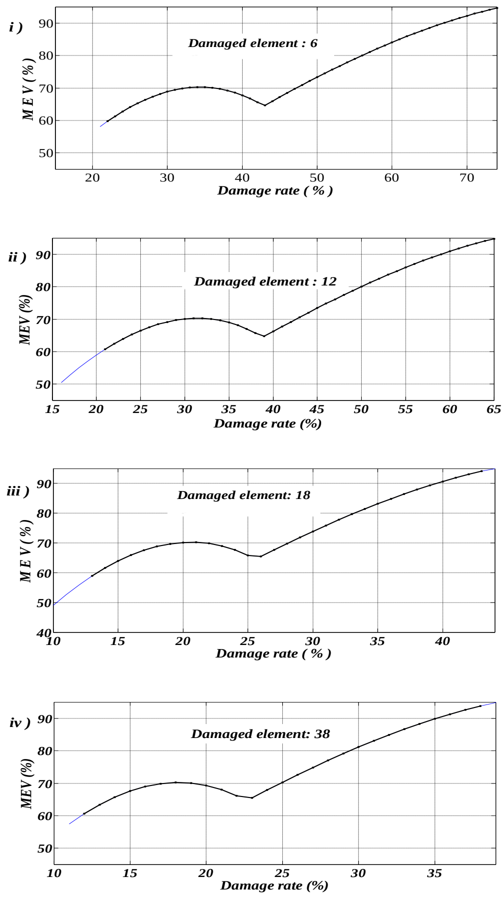


FIG. 4 – Simply-simply supported beam : MEV histogram in terms of 40% damaged beam element number



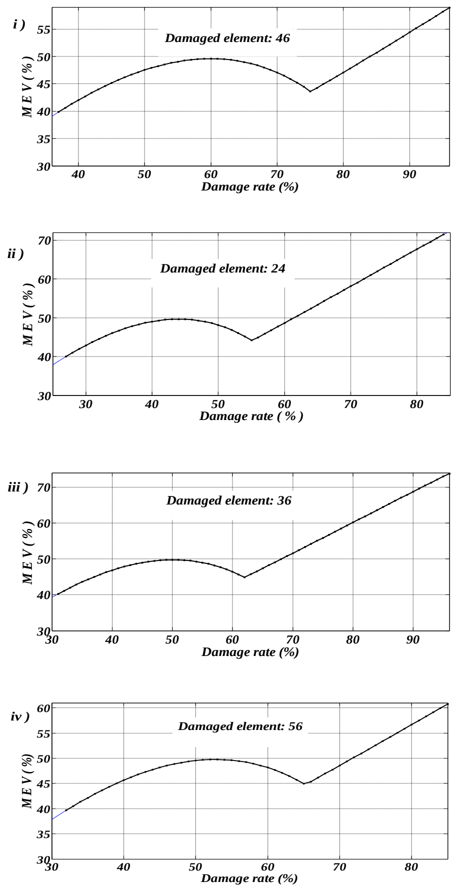


FIG.5a - : Cantilever beam FIG.5*b* : Simply-simply supported beam

FIG. 5 –Variation of MEV according to the damage rate.

On FIG. 4 we choose element 46 and seek the good combination of the components of the exciting force giving the representative MEV curve. The threshold is fixed here at 40% and the smallest damage is located at approximately of 40%.

For all the structures the threshold of MEV is fixed at 40%. Of each curve we take note of the minimum detectable rate of damage. Some curves are presented in FIGS.5.b. The threshold of detectability is 35% for element 6; 38% for element 10; 44% for element 12 and 46 for the 56ème damaged element.

FIG.5 give us detailed information about the variation of MEV according to the size of the defect for each case of structure with different damaged element, and help us locate the threshold of detectability for each one of them. We notice a correspondence between the data of the histograms and those of FIGS.5, i.e. the elements having low values of MEV on the histograms have a detectability threshold higher and vice versa. For example, on histogram 3, the value of MEV for element 12 is 20% and for element 28 it is 35%. On FIG.5, the damage is detectable from the value of MEV of 12% if the damaged element is the 38ème while it is detectable only from a MEV value of 20% for the élément12. The application of the method on the two types of boundary conditions gives us similar results.

6 DAMAGE DETECTION-LOCALIZATION METHOD

6.1 PRESENTATION OF THE METHOD OF DETECTING AND LOCALIZING DAMAGE IN STRUCTURES.

Let $y(t)$ be the dynamic response signal of the structure rebuilt after wavelet packet decomposition, according to formula (6). Its total energy U is given by the formula (8). Total energy structures, healthy and damaged, are designated by U^h and U^d respectively.

First, we define $VEPO_{nq}$ by the absolute value of the relative change between the total energy of the healthy structure U^e and that of the damaged structure U^e, measured from the nq^{th} node.

$$VEPO_{nq}(\%) = \frac{\left|U^h - U^d\right|}{U^h} = \frac{\left|\sum_{j=1}^{2^k} U_k^{j(h)} - \sum_{j=1}^{2^k} U_k^{j(d)}\right|}{\sum_{j=1}^{2^k} U_k^{j(h)}} \times 100\% \qquad (12)$$

where U_k^j is the energy of the j^{th} sub-signal in the decomposition of the structure of the response by the k^{th} wavelet packet level.

The response of the structure is measured in a single DOF. The principle of this procedure is to calculate the $VEPO_{nq}$ for two DOF belonging to the same element, and calculating the relative difference between them.

Among the 20 DOFs of the SI20 finite element, we chose the two vertical DOFs of the ends of each element to measure the $VEPO_{nq}$. The VEPO value estimated at the first DOF defining element q is affected by the index q and the second by the index $q+1$ (Fig. 6).

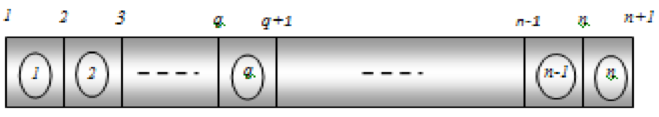


Fig. 6. Numbering of vertical DOF of the structure

We then calculate, for each q element, an IVEPO indicator (Energy change indicator based on Wavelets Packets) defined by:

$$IVEPO(\%) = \left|\frac{VEPO_{nq+1} - VEPO_{nq}}{VEPO_{nq}}\right| \times 100(\%) \qquad (12)$$

The application of our indicator for fault location on a structure requires as many sensors as the number of discretization elements. This, from a practical point of view, is a cumbersome burden for the structure and causes expensive implementation costs. An alternative to these constraints is to use a method for optimizing the number of sensors used and their positioning on the structure. In this work, the method of H. Amdriambolona [3] for selecting DOF sensors and reconstituting the unobserved DOF is used.

6.2 NUMERICAL SIMULATION TEST CASE

We consider the composite beam structure of graphite/epoxy in Figure 7 having the configuration. It is discretized into 40 finite elements SI20, numbered 1 to 40 from the left to the right.

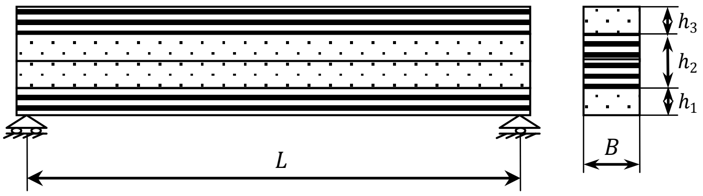


Fig. 7. Doubly simply supported layered $[0°/(90°)_2/0°]$ beam.

The mechanical characteristics of the beam material are given by:
E1 = 144,8GPa; E2 = 9,65GPa; G13 = 4,14GPa; G23 = 3.45 GPa; μ12 = 0.25; ρ = 1390 kg / m3.
Its geometry is characterized by:
L = 15m; h1 = h3 = 0.25m; h2 = 0.5 m; B = 1m.
In the numerical simulation tests that follow, we examine the sensitivity of the indicator in the case of the presence of two and three defects.
In all tests, we adopt the same approach:

- the simulation of damage to a component is performed by a 20% reduction of the longitudinal Young's modulus of the second layer (E2) and 5% of its transverse shear modulus (G23).
- we excite all the structures in the same position (vertical excitation in the 20th node from the right end) with a single exciting force of the form. The dynamic response is calculated for the two structures, healthy and damaged.

Among the 40 vertical translational DDL, we selected 23 sensors positions. For the simulation, we added random noise to simulate responses of both healthy and damaged structures using model: YBT y = (1 + gn × rand).
Then, the responses of healthy and damaged structures are then decomposed by wavelet packet of order Daubechies 4 "db4" the 5th level. Then we calculate the IVEPO for each element and we represent IVEPO-number of the element of each test curves.

A. Case of two damages

We examine three cases of structures: each containing two damaged components: The first structure has its elements 23 and 24 damaged; the second is damaged in the elements 11 and 33 and the third is damaged in elements 6 and 37. Representative histograms are shown in Figures above.

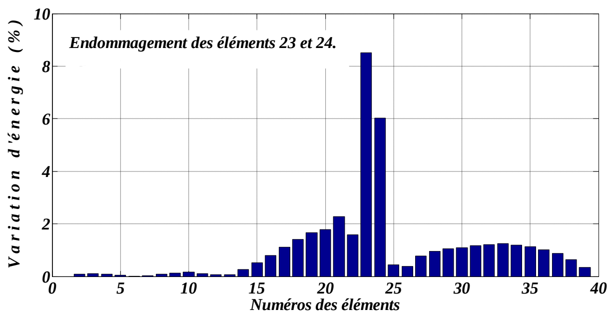


Fig.8. Histogram IVEPO-number of the element in the case of damaged items 23 and 24

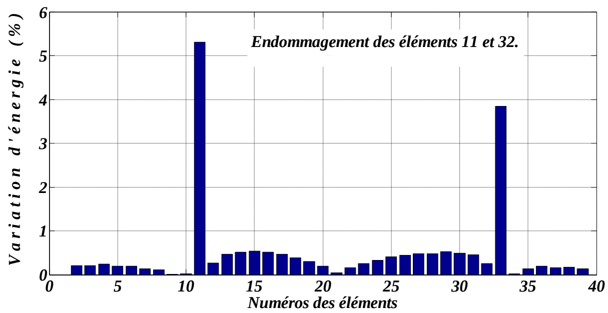


Fig.9. Histogram IVEPO-number of element in the case of damage to the elements 11 and 32

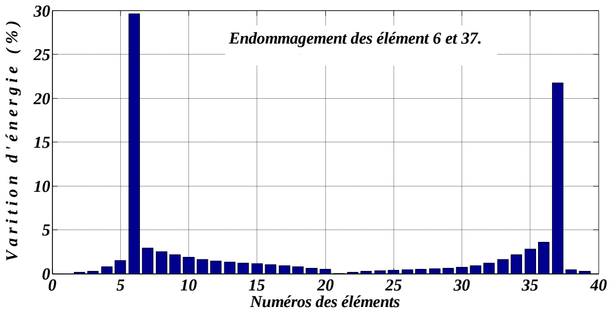


Fig.10. Histogram IVEPO-number of element in the case of damage to the elements 6 and 37

B. Case of three damages

As in the first case, we simulate three damaged structures having each three damages. For the first, it is damaged in its elements 10, 13 and 22; the second, items 7, 10 and 35, and finally the third, we simulate damage to the 11th, 14th and 32nd element. The results of these tests are shown in the following figures.

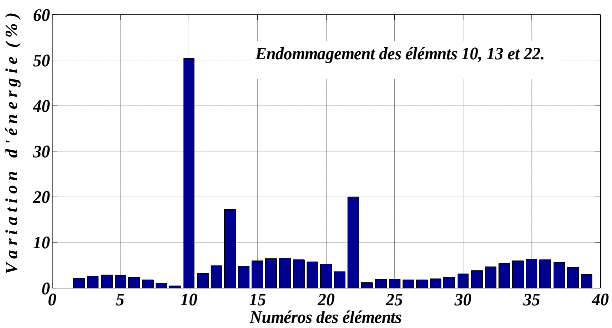


Fig. 11. Histogram IVEPO-element number in the case of damage to elements 10, 13 and 22

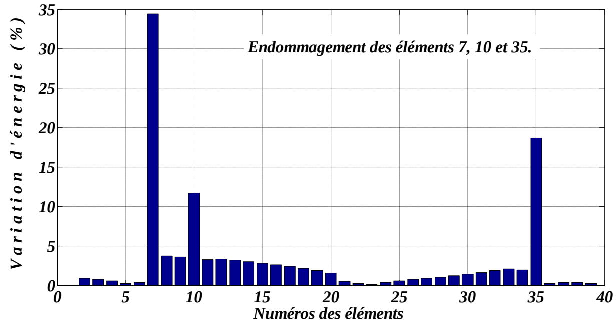


Fig. 12. Histogram IVEPO-number in the case of damage to elements 7, 10 and 35

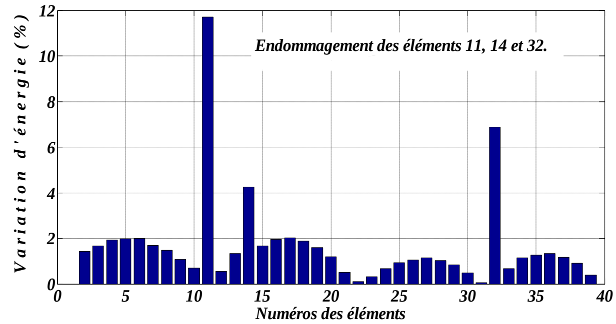


Fig. 13 Histogram IVEPO-element number in the case of damage to elements 11, 14 and 32

7 CONCLUSION

In this work, we propose a damage indicator based on the decomposition of wavelet packet responses to locate damage in composite beam structures in their first stage of development. This indicator is the relative difference between the total energy of the sub-signals of the healthy structure response and that of the corresponding ones of the damaged structure response. A resembling indicator proposed by Jian-Gang Han [4] is defined differently. The latter is equal to the sum of all relative energy differences of corresponding sub-signals of healthy and damaged responses included in the wavelet packets decomposition.

The property of wavelet packets decomposition for denoizing signals is certainly of great help particularly in the case of laminated composite structures.

As for the global threshold damage indicator is concerned, our contribution consists in finding a global threshold damage indicator rather than a particular one for a defined position of the damaged element in the structure as did Yam and al. This global threshold damage indicator is useful to help ascertain whether results of the localization process are reliable or not.

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