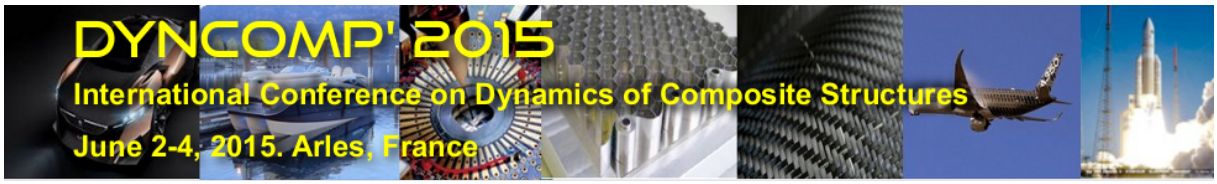

ROBUST DECISION MAKING: AN APPLICATION OF INFO-GAP METHOD ON A DYNAMIC MODEL

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ABSTRACT

The aeronautics industry have benefited from the use of numerical models to supplement or replace the costly design-build test paradigm. Dynamic models are commonly calibrated to obtain optimal fidelity to data. In order to improve calibration methods, we wish to take into account the unavoidable compensating effects between the parameters which lead to the non-uniqueness of the responses. An envelope-bound info-gap model will be used to explore the change in predictions as the the parameter values are allowed to vary for different horizons of uncertainty. The calibration is considered robust if an acceptable level of fidelity to data is obtained even in the presence of uncertainty. Our methodology is demonstrated on an example involving the dynamic response of a clamped plate with progressively reduced thickness.

1 INTRODUCTION

Model calibration methods improve the correlation between finite element models (FEM) and measured data. The aim is to obtain the most predictive analytical model despite their incompleteness to describe exactly the underlying physics: most of the parameters should be considered as uncertain rather than nominal values due to manufacturing and experimental variability. In this case, a stochastic calibration method should be use.

On the other hand, the calibration convergence can be wrong due to physical compensating effects which lead to fidelity-equivalent solutions. The info-gap theory provides a way to ensure that the system remains reliable even under these unknown compensating effects. This paper presents an approach to enhance the robustness of a stochastic calibration method using the info-gap theory.

2 ROBUST CALIBRATION

2.1 Calibration performances

We wish to compare experimental data results to simulatated outputs. A common metric to evaluate deterministic calibration performances is the normed Euclidean distance D_E:

$$D_E = \sum_{i=1}^{n} \frac{\sqrt{(v_{a_i} - v_{m_i})^2}}{v_{m_i}} \tag{1}$$

v_m is a vector containing the nominal eigenfrequencies measured (n ouputs) and v_a the corresponding vector containing the nominal analytical responses and the same number of outputs.

Parameters may be considered as uncertain and defined by probability density functions. In this case, the model now provides uncertain outputs and can be calibrated using stochastic approaches such as covariance adjustment [**?**], Gibbs sampling [**?**] and Metropolis-Hasting algorithm [**?**]. The Euclidean distance is not suitable to compare two unknown distributions whereas Bhattacharya distance D_B is relevant to evaluate multivariate features [**?**]. This metric takes into account both the mean-difference and the covariance difference between the two distributions :

$$D_B = \frac{1}{8}(\bar{v}_a - \bar{v}_m)^T \Sigma^{-1} (\bar{v}_a - \bar{v}_m) + \frac{1}{2} \ln\left(\frac{\det(\Sigma)}{\sqrt{\det \Sigma_a \det \Sigma_m}}\right) \tag{2}$$

with $\bar{v}_m$ the vector containing the measured eigenfrequencies mean values and $\bar{v}_a$ the mean vector of the mean analytical responses. The pooled matrix Σ is given by the combination of Σ_m the covariance of the experimental eigenfrequencies and Σ_a the covariance of the analytical ouputs as $\Sigma = \frac{\Sigma_a + \Sigma_m}{2}$.

2.2 Info-gap theory

Info-gap theory has its origins in Ben-Haim [2] studying the reliability of mechanical systems. Since, this approach has been use on a wide range of applications such as climate models [3] and medical researches [4]. The purpose of info-gap is to provide tools for decision-makers in order to assess risks and opportunities of a model in light of the analysis of severe lack of information.

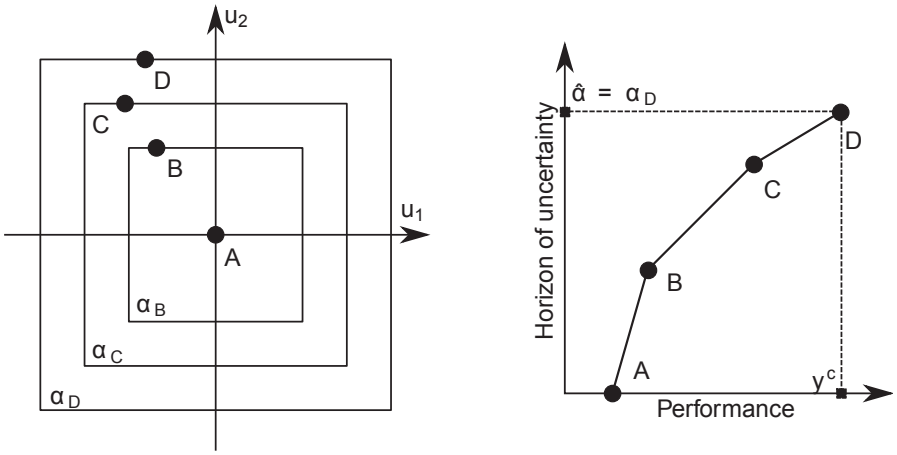


Figure 1. Nested subsets (left) - Robustness curve (right).

In practice, important modeling information may be lacking due to an incomplete understanding of the underlying physics. Hence, probability density functions are not generally suitable alone to describe severely uncertain parameters θ.

We introduce the horizon of uncertainty characterized by α. The larger α, the greater the range of unknown parameter variations i.e. a bigger space is available for uncertain parameters. Numerical model performances are commonly defined by a function which assess the quality of the response fidelity. Consequently, a catastrophic failure may appear for one set of uncertain parameters sampled from the previous space.

Let's consider several horizons of uncertainty α_i, the function that yields the worst case model predictions for a given horizon α is called $\hat{R}$ and calculated as follow [5] :

$$\hat{R}(\alpha) = \max_{\theta \in U(\alpha, \tilde{\theta})} R(\theta) \tag{3}$$

$\tilde{\theta}$ is the calibrated best-estimate parameter values of the simulation model. The robustness function expresses the greatest level of uncertainty at which performance remains acceptable.

$$\hat{\alpha} = \max\{\alpha : \hat{R}(\alpha) \geq R_c\} \tag{4}$$

with $\hat{\alpha}$ the maximum horizon to which info-gap uncertainty model is allowed to expand as long as minimal requirements R_c are satisfied.

The figure 1 explains in a schematic way the method to compute robustness curve [6]. The unknown parameters are u_1 and u_2. At the first step α_A, we consider no uncertainty thus the space contains a single point A which is necessarily the worst case. The respective performance $R(u_{1A}, u_{2A})$ is noted on the robustness curve on the right. Second step, the horizon of uncertainty is increased to α_B. In the space defined by α_B, the worst case can be found using a factorial design or by optimization [7]. The algorithm returns the worst case B and report it on the robustness curve. The procedure can be repeated for as much nested subsets required.

3 NUMERICAL APPLICATION

A new generation of ceramic matrix composite (CMC) turbine blades have been developed [8]. These materials show high resistance to extremely high temperature ($1000°$C), low density and a good fracture toughness compared to conventional metallic alloys.

The approach developed previously is applied to a CMC plate perfectly clamped with a progressively reducing thickness. The plate is itself divided into three isotropic material parts to represent physical heterogeneity due to the industrial process. Differences between the experimental results and the numerical outputs will be analyzed through the first three eigenfrequencies of the structure.

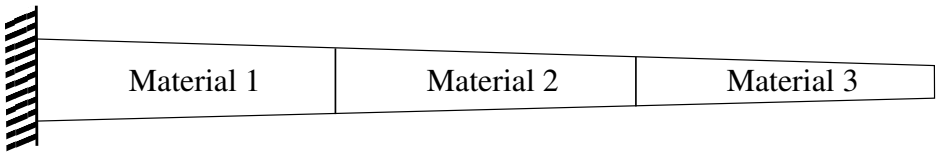


Figure 2. Material properties distribution.

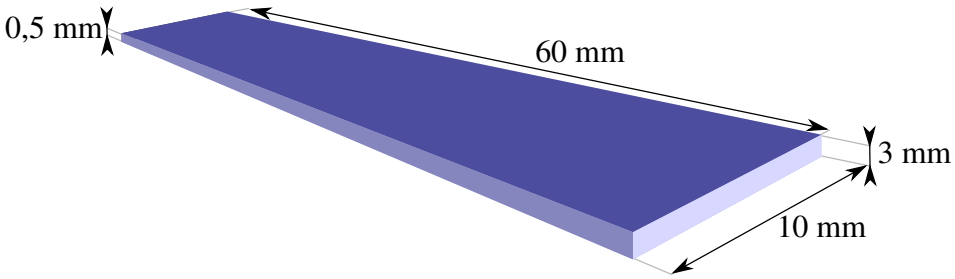


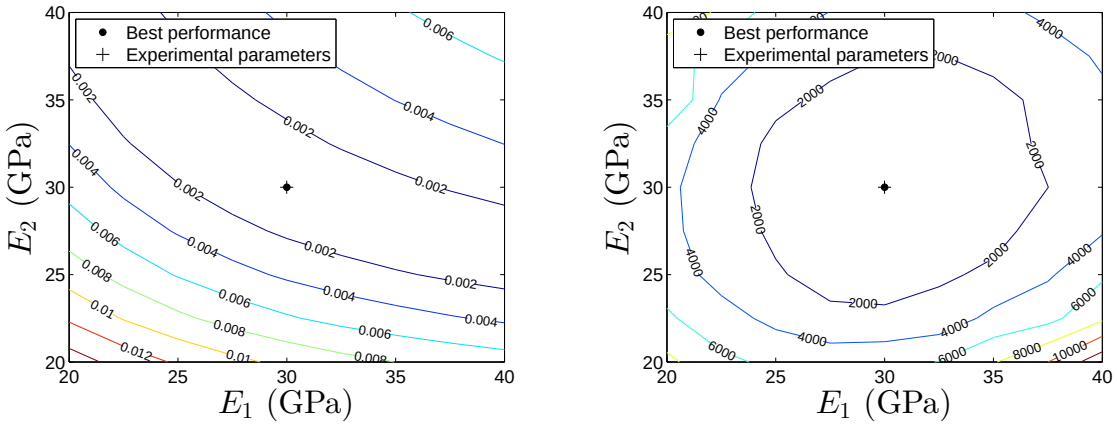
Figure 3. Plate dimensions.

The system is studied through material 1 and material 2 Young's Modulus, noted respectively E_1 and E_2. The range of parameters E_1 and E_2 values is [20 GPa;40 GPa], divided into a 20×20 grid. Thus, 300 Monte-Carlo sampling are achieved with NASTRAN for each combination of parameters using these values as mean. The parameters variance are defined as 10% of the corresponding mean values.

For the simulated test data, 500 experiments are sampled in the same way with $E_1 = E_2 = 30$ GPa and their variances $\sigma_{E_1} = \sigma_{E_2} = 3$ GPa. No model form error is added between the two samples. The nominal exact outputs are $\nu_1 = 18.15$ Hz, $\nu_2 = 50.64$ Hz and $\nu_3 = 78.25$ Hz.

The error surface responses are plotted in the space of the two parameters (Figure 4). The contours illustrate fidelity-equivalent solutions and define satisfying boundaries. The best performance marker stands for the global minimal distance found in the discrete space. As expected, this marker totally coincides with the experimental parameters marker for both distance metrics. In this case, the corresponding couple of parameters provides exact and optimal solutions.

In Figure 4(a), there is a slender space where the error remains below 0.2%. It means that deterministic calibrating algorithms can find acceptable set of parameters quite far from the actual experimental parameters. These compensating effects are inevitable even in the absence of bias in the model prediction. In Figure 4(b), the isocontours create nested circles which the lowest error in located in the center. In this case, stochastic calibration should provide relevant updated parameters.



(a) Euclidean isocontour.

(b) Bhattacharya isocontour.

Figure 4. Euclidean and Mahalanobis distances.

4 CONCLUSION

This paper proposes a framework to motivate the robust calibration process. In the case study, material parameters have been considered uncertain and the compensating effects between them as a lack of knowledge. Successful updating provides parameters which minimize the error between simulated experiments and analytical outputs while taking into account unavoidable compensating effects. Two calibration metrics have been investigated, in particular a deterministic euclidean error as well as the statistical Bhattacharyya error.

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