SENSITIVITY ANALYSIS OF TRANSMISSION LOSS THROUGH COMPOSITES WITH ACOUSTIC TREATMENT

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ABSTRACT

Composite materials are widely used in the aerospace industry, for their low mass and high stiffness, however, these characteristics tend to increase noise transmission. Sound protection must therefore be added, in the form of porous material layers. Uncertainties may affect both the structural and sound package parameters. It is therefore important to assess the influence of these uncertain parameters on the sound transmission properties of the assembly. The sound transmission loss through a composite plate-foam assembly is first computed with the transfer matrix method. The effect of uncertainty of several parameters such as the porosity, flow resistivity and mechanical parameters is then analysed with the FAST (Fourier amplitude sensitivity test) method. The effect of adding a thin screen at the interface between the porous and air is also investigated.

1 INTRODUCTION

Noise transmission is often a major concern in the industry. Composite structures are known to have lower acoustic performance than their metallic counterparts, but their high stiffness to mass ratio makes them more and more used in aerospace applications. On of the most frequently used construction is the sandwich one, with stiff skins constraining a softer, shearing core. Analytical models of sound transmission have been proposed in the literature [1, 2]. Some kind of acoustic treatment is then needed to enhance the transmission loss. Porous materials such as fibreglass are commonly employed for this purpose, which can be modelled with the Biot model [3, 4]. Sometimes a thin screen can be glued to the porous material to protect it on the transmission side.

Some variability always occur when modelling the transmission loss of structures with noise treatment, due either to uncertainty in the parameter measurement, or to design latitudes allowing for optimisation. It is therefore of utmost importance to assess the effect of this variability on the model output, and estimate the sensitivity of it with respect to each uncertain parameter. Several methods have been proposed for this purpose, one of the most popular being the evaluation of Sobol indices [5]. The Fourier Amplitude Sensitivity Test (FAST) method [6]has been proposed to accelerate the computation of these indices and already used successfully for acoustic and poroelastic applications [7].

We propose here to use the (FAST) method to investigate the effect of several parameters of a plate-porous assembly such as the one shown on figure 1. This paper is structured as follows. The FAST method is first presented in section 2. A model of transmission loss through infinite plane assemblies of composite a porous materials based on the transfer matrix method is presented in section 3. Finally some results are discussed in section 4.

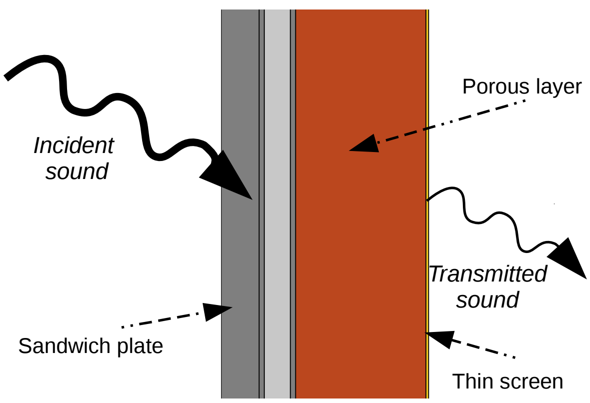

Figure 1: View of the studied configuration. The influence of the presence of a limp screen on the transmission side is studied

2 GLOBAL SENSITIVITY ANALYSIS: THE FAST METHOD

In the analysis of variance technique, a parameter's influence on the model output is quantified by the impact it has on the variance in the given design range. In the following development, a generic mathematical model is considered. A model is a real valued function f defined over K^n, where $K = [0, 1]$. With appropriate scaling and translations, any model defined over continuous ranges of parameters can be represented that way.

For a given model f linking input parameters $\mathbf{x} = (x_1, ...x_n)$ to a scalar output $y = f(\mathbf{x})$, there exists a unique partition of f so that

$$y = f(x_1, x_2, ...x_n) = f_0 + \sum_{i=1}^{n} f_i(x_i) + \sum_{i<j} f_{ij}(x_i, x_j) + ... + f_{1...n}(x_1, ..., x_n) \quad (1)$$

provided that each function f_I involved in the decomposition has zero mean over its range of variation. The decomposition given by equation 1 is called the Hoeffding decomposition or high order model representation (HDMR) [8].

For a given set of indices $I = \{i_1, \ldots, i_n\}$, the partial variance is therefore the variance of f_I

$$D_I = \int_{K|_I} f_I(x_I)^2 \mathrm{d}x_I \quad (2)$$

the sensitivity index relative to the set I is expressed as the ratio of the variance of the function f_I to the total variance of the model:

$$\mathrm{SI}(I) = \frac{D_I}{D}. \quad (3)$$

The computation of all the 2^n sensitivity indices is needed to represent completely the model, however this becomes quickly a very costly task in terms of computational time, as they have to be evaluated by numerical integration. However, most information about a parameter's influence can be found in the first-order sensitivity index and the total sensitivity index, which can be computed more efficiently with the FAST method.

For a given parameter $i \in [1, n]$, the main effect (ME) is then the sensitivity index relative to the 1-dimensional function f_i.

The first-order index represents the share of the output variance that is explained by the considered parameter alone. Most important parameters therefore have high ME, but a low ME does not mean the parameter has no influence, as it can be involved in interactions.

The idea of the FAST method is to avoid the evaluation of the n-dimensional integrals needed for the computation of the f_i functions, and replace them by a single 1-dimensional integral along a *space-filling* curve in the design space. This curve is defined so as to be periodic with different periods relative to each parameter. Saltelli [9] propose the sampling function defined by:

$$x_i = \frac{1}{2} + \frac{1}{\pi} \arcsin\left(\sin\left(\omega_i s + \varphi_i\right)\right) \quad (4)$$

The frequencies ω_i are integers chosen so as to minimize interference between parameters[10]. The frequencies are said to be free of interference up to order M if all linear combinations

$$\sum_{i=1}^{n} \alpha_i \omega_i \neq 0 \quad (5)$$

where $\alpha_i \in \mathbb{Z}$ and $\sum_{i=1}^{n} |\alpha_i| < M$.

As all frequencies are integers, the resulting function is 2π-periodic with respect to variable s. The sampling is then done using $N > 2\omega_n + 1$ samples in the $[0, 2\pi]$ interval. Calling $y_k = f(x_k)$ the model output on each sample, the discrete Fourier transform $\hat{y}_k$ can be computed.

The total variance of the function in the design space is computed with Parseval's theorem as

$$D = \int_K f^2(x) - f_0^2 \mathrm{d}x \approx \sum_{k=1}^{N} y_k^2 = \sum_{k=1}^{N} \hat{y}_k^2 \quad (6)$$

The contribution of parameter i will then be:

$$D_i = \sum_{k=1}^{M} \hat{y}_{k\omega_i} \tag{7}$$

3 THE SIMPLIFIED TRANSFER MATRIX METHOD

The sound transmission loss through a multilayered structure composed of plates, air gaps and poroelastic materials can be computed with the transfer matrix method (TMM). This method was first proposed by Brouard *et al.* [11] and extended by Allard and Atalla [4]. We shall use here a simplified version presented by Hu [12] valid for limp poroelastic materials. The acoustic behaviour of the structure can be represented with only the fluid pressure p and normal velocity v as state variables. These two variables are defined in each point in the fluid layers, and on each side of the solid layers. Each layer can be represented by a 2×2 matrix linking the state variables on one side to those on the other side, and a global transfer matrix can be obtained by multiplying all these matrices together. The transfer equation then reads:

$$\begin{pmatrix} p_L \\ v_L \end{pmatrix} = \begin{pmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{pmatrix} \begin{pmatrix} p_R \\ v_R \end{pmatrix}, \tag{8}$$

where indices R and L stand for right and left sides of the structure. Waves on each side can be decomposed in positive- and negative-going waves, which can be written :

$$p_R = p_R^+ + p_R^- \qquad \text{and} \qquad p_L = p_L^+ + p_l^-. \tag{9}$$

According to the pressure-displacement relation in the fluid, the normal velocities are:

$$v_R = Y_0(p_R^+ - p_R^-) \qquad \text{and} \qquad v_L = Y_0(p_L^+ - p_l^-), \tag{10}$$

where $Y_0 = \cos\theta/\rho_0 c_0$ is the characteristic admittance of the surrounding fluid.

This leads to rewriting equation 8 as

$$\begin{cases} p_L^+ + p_L = T_{11}(p_R^+ + p_R^-) + Y_0 T_{12}(p_R^+ - p_R^-) \\ p_L^+ - p_L = \dfrac{T_{21}}{Y_0}(p_R^+ + p_R^-) + T_{22}(p_R^+ - p_R^-) \end{cases}. \tag{11}$$

We will be studying transmission of a plane wave incident from the left side, whose interaction with the structure creates a reflected wave into the left side, and a transmitted wave into the right side. In that case, no negative-going wave will propagate in the right side. The acoustic transparency is defined as the ratio of transmitted to incident acoustic intensities, which reduces to

$$\tau = \left| \frac{p_R^+}{p_L^+} \right|^2 \tag{12}$$

in the case of identical fluids on each side of the structure. Solving the system in equation 11, we get

$$\tau(\omega, \theta) = \frac{1}{4} \left| T_{11} + T_{12} Y_0 + \frac{T_{21}}{Y_0} + T_{22} \right|^2. \tag{13}$$

The diffuse field transmission loss is then obtained by performing a weighted average of the transparency over an angular range. The full range [0;$\pi/2$] is retained here. In that case, we get the diffuse field transparency:

$$\tau_d(\omega) = 2 \int_0^{\pi/2} \tau(\omega, \theta) \sin\theta \cos\theta \mathrm{d}\theta. \tag{14}$$

The transmission loss (TL) is then defined as

$$\mathrm{TL} = -10\log_{10}\tau_d. \tag{15}$$

The transfer matrices for a sandwich plate and a limp poroelastic material are derived in the following subsections. Due to the forced nature of the excitation, the transverse wavenumber $k_t = \frac{\omega}{c}\sin\theta$ and the pulsation of the incident wave ω are conserved across the whole system.

3.1 Transfer matrix of a sandwich plate

The transfer matrix of a general plate can be obtained from its constitutive equation in presence of forced loads. When excited by a plane wave with frequency ω, the plate will vibrate and radiate one acoustic wave on each side, respectively reflected and transmitted. The constitutive equation can be put under the general form

$$Zv = p_L - p_R, \tag{16}$$

where Z, a linear operator, is the impedance of the plate. The sound field on the left side of the plate is p_L and on the right side it is p_R.

The continuity of normal speed between the surrounding fluids and the plate imposes $v_L = v_p$, hence

$$\begin{pmatrix} p_L \\ v_L \end{pmatrix} = \begin{pmatrix} 1 & Z \\ 0 & 1 \end{pmatrix} \begin{pmatrix} p_R \\ v_R \end{pmatrix} \tag{17}$$

For a sandwich plate, the constitutive equation is given by Mead [1], with five main parameters, namely skin bending stiffness D_t, overall bending stiffness B, damping η, surface mass m and shear stiffness g. After minor corrections, this reads:

$$D_t(1+i\eta)\nabla^6 w - g(D_t+B)(1+i\eta)\nabla^4 w + m\frac{\partial^2 w}{\partial t} - \frac{mB}{N}\frac{\partial^2}{\partial t^2}\nabla^2 w = (\nabla^2 - g)(p_L - p_R), \tag{18}$$

where $w = v/i\omega$ is the normal displacement of the plate. In the considered frame where a forced wave is imposed on the plate with a wavenumber $k_t = \frac{\omega}{c}\sin\theta$, the spatial derivative operator ∇ can be replaced by $-ik_t$. This leads to the following expression of the impedance

$$Z(\omega,\theta) = \frac{D_t k^6 + g(D_t+B)k^4 - m\omega^2 k^2 - mg\omega^2(1-\nu^2)}{i\omega(k^2+g)}. \tag{19}$$

For sandwiches made of isotropic materials and identical skins, the skin bending stiffness is $D_t = \frac{Eh_s^3}{6(1-\nu^2)}$ the overall bending stiffness is $B = Eh_c^2 h_s(1+\frac{h_s}{h_c})^2/2$ and the shear stiffness is $g = Gh_c\left(1+\frac{h_s}{h_c}\right)^2$. This expression is equivalent to that of a thin plate if the shear stiffness is infinite.

3.2 Limp poroelastic model

Poroelastic materials can be modelled with the Biot-Allard model, taking into account wave propagation in the fluid and solid phases. However, if the material is especially limp, like fibreglass, it can be possible to neglect the solid part and model it as an equivalent fluid with complex and frequency dependent parameters. The wave propagation equation in the porous layer reduces to one scalar equation[4]

$$\Delta p + \frac{\tilde{\rho}_{limp}}{\tilde{K}_{eq}}\omega^2 p = 0, \tag{20}$$

where $\tilde{\rho}_{limp}$ is the equivalent density and $\tilde{K}_{eq}$ the equivalent bulk modulus of the fluid representing the porous material. These two quantities are complex and frequency dependent. Their expression is given in chapter 5 of reference [4]:

$$\tilde{K}_{eq} = \frac{\gamma P_0}{\phi\left(\gamma - \frac{\gamma-1}{K}\right)} \tag{21}$$

$$\tag{22}$$

$$\tilde{\rho}_{limp} = -\frac{\rho_0^2 - \frac{1}{\phi^2}(\rho_1 + \phi\rho_0)(\frac{iB}{\omega} + \alpha_\infty\phi\rho_0)}{\rho_1 - 2\rho_0 + \phi\rho_0 + \frac{1}{\phi^2}(\frac{iB}{\omega} + \alpha_\infty\phi\rho_0)} \tag{23}$$

where the coefficients K and B can be expressed as

$$K = 1 + \frac{8\mu_0}{i\omega \mathrm{Pr}\Lambda_{therm}^2\rho_0}\sqrt{1 + i\omega\frac{\mathrm{Pr}\Lambda_{therm}^2\rho_0}{16\mu_0}} \tag{24}$$

$$\tag{25}$$

$$B = \sigma\phi^2\sqrt{1 + 4i\omega\frac{\alpha_\infty^2\mu_0\rho_0}{(\sigma\Lambda_{visc}\phi)^2}}. \tag{26}$$

The parameters Pr, μ_0, P_0 and ρ_0 are respectively the Prandtl number, the dynamic viscosity, the bulk modulus and the density of air, whose reference values at 20^oC are given in table 1.

The porous material is described by six characteristic parameters, namely the porosity ϕ, the flow resistivity σ, the static tortuosity α_∞, the viscous and thermal dissipation characteristic lengths Λ_{visc} and Λ_{therm}, and the *in vacuo* skeleton density ρ_1.

Parameter	description	unit	value
Pr	Prandtl number	-	0.71
μ_0	dynamic viscosity	Pa.s	$1.845 \cdot 10^{-5}$
ρ_0	density	$\mathrm{kg.m}^{-3}$	1.21
P_0	reference pressure	Pa	101325

Table 1. Reference parameters for air at 20^oC.

The complex wavenumber of the wave propagating in the equivalent fluid is, according to equation 20:

$$k = \omega\sqrt{\frac{\tilde{\rho}_{limp}}{\tilde{K}_{eq}}}, \tag{27}$$

and the normal component is $k_n = \sqrt{k^2 - k_t}$.

The transfer equation between two points inside the equivalent fluid separated by a distance h then writes:

$$\begin{pmatrix} p_L \\ v_L \end{pmatrix} = \begin{pmatrix} \cos(k_n h) & i\omega\frac{\tilde{\rho}_{limp}}{k_n}\sin(k_n h) \\ i\frac{k_n}{\omega\tilde{\rho}_{limp}}\sin(k_n h) & \cos(k_n h) \end{pmatrix} \begin{pmatrix} p_R \\ v_R \end{pmatrix}. \tag{28}$$

The previous equation is valid for the wave inside the fluid. When coupled to another medium, the continuity of normal speed should account for the porosity of the material. If the other material is a plate or the surrounding air, this conditions reads

$$\phi v_{poro} = v_m, \tag{29}$$

where v_m is the normal velocity inside the other medium. The complete transfer matrix of the porous layer then writes:

$$T_{poro} = \begin{pmatrix} 1 & 0 \\ 0 & \phi \end{pmatrix} T_p \begin{pmatrix} 1 & 0 \\ 0 & \frac{1}{\phi} \end{pmatrix}. \tag{30}$$

4 RESULTS

The transmission loss of a system composed of a honeycomb panel and a fibreglass layer has been studied. This was modelled with the analytical model described in section 3, where the global transfer matrix is

$$T_{bare} = T_{sandwich} T_{poro}, \tag{31}$$

where $T_{sandwich}$ is given in equation 17 and T_{poro} in equation 30. A second configuration in which a thin limp screen is glued to the porous layer at the interface with the receiving cavity is studied. In this case, the transfer matrix is

$$T_{screen} = T_{bare} \begin{pmatrix} 1 & i\omega m_{screen} \\ 0 & 1 \end{pmatrix}. \tag{32}$$

In both cases, the diffuse field transmission loss is computed with equation 15 between 100Hz and 10kHz. All constant parameters are summarised in table 2. We would like to study the effect of five parameters on the overall transmission loss in the two configurations, namely 2 parameters of the sandwich, core shear modulus and damping coefficient, and 3 parameters of the fibreglass layer, its porosity ϕ_0, flow resistivity σ and viscous characteristic length Λ_{visc}. The two characteristic lengths Λ_{visc} and Λ_{therm} are usually correlated, which will be taken into account by taking $\Lambda_{therm} = 2\Lambda_{visc}$. The variation ranges of these five parameters are shown in table 3. They are chose as realistic considering both uncertainty in measurement and some latitude in design.

Results of the FAST analysis are presented in figure 2 for the bare case and 3 for the case with a screen. The sensitivity indices of each parameter are presented as proportions of the standard deviation. Some conclusions can be drawn for both cases: none of the considered parameters is important in low frequency, while the dominant parameter in HF is the viscous length, which accounts also for the thermal characteristic length, as they are considered proportional. The parameters of the structure (G and η) have no significant incidence on the transmission loss in their considered variation ranges. This is due to the fact that the considered frequency range is well below the coincidence frequency, which occurs around 19kHz.

The transmission loss variation range is shown for the two cases in figure 4 for the two cases. It can be seen that the addition of a thin screen reduces the loss in low frequency, but improves in much more in high frequency. A mass-fluid-mass resonance phenomenon appears in both cases, where the TL is lower around 500Hz for the screen case, and around 1kHz for the bare case. In both cases, flow resistivity σ is the dominant parameter between 1000 and 1500Hz. This phenomenon is due to the mechanical resonance of the cavity filled of porous, analogous to what happens in a double-plate system.

The main difference between the two cases in terms of sensitivity is the preponderance of porosity between 400 and 1200 Hz when a screen is placed after the porous material. The overall effect of the screen is globally to increase the transmission loss above 650 Hz, and reduce the variability of the TL with respect to the investigated parameters.

5 CONCLUSION

A model of transmission loss through composite sandwich plates with attached limp poroelastic materials based on the transfer matrix method has been proposed in this paper. Its sensitivity

Parameter	description	unit	value
E	Skin Young modulus	GPa	47
ν	Poisson ratio	–	0.1
h_{skin}	Skin thickness	mm	1
h_{core}	Core thickness	mm	12.7
m	Sandwich surface density	$\mathrm{kg.m^{-2}}$	8
α_∞	Tortuosity	–	1.25
ρ_1	Porous *in vacuo* density	$\mathrm{kg.m^{-3}}$	5.5
Λ_{therm}	Thermal characteristic length	μm	$2\Lambda_{visc}$
h_{poro}	Porous thickness	mm	50
m_{screen}	Screen surface density	$\mathrm{kg.m^{-2}}$	0.2

Table 2. Constant parameters considered in this study

Parameter	description	unit	min. value	max. value
G	Shear modulus of the sandwich's core	MPa	20	40
η	Structural damping	–	10^{-3}	10^{-2}
ϕ_0	Porosity	–	0.85	0.99
σ	Flow resistivity	$\mathrm{kN.m^{-4}.s}$	10	30
Λ_{visc}	Viscous characteristic length	μm	25	75

Table 3. Variable parameters considered in this study

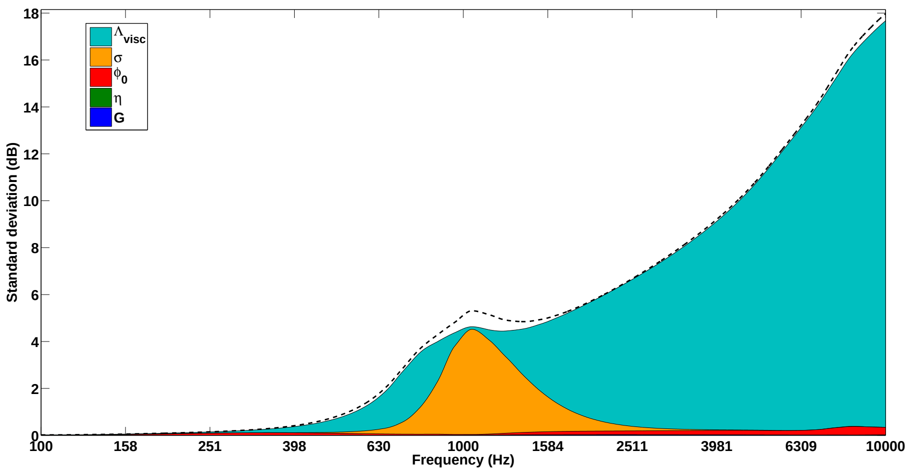

Figure 2. Sensitivity indices in the bare configuration

to several parameters is studied with the FAST method, which allows to efficiently estimate the sensitivity indices of parameters subjected to uncertainty in a model. However the uncertainty level of the parameters should be known before the analysis, in the form of a variation range or a probability distribution.

In the considered case, it has been found that the most important parameters in high frequency are the viscous and thermal characteristic lengths, as well as the flow resistivity in an intermediate frequency range around the mass-fluid-mass resonance. The presence of a light thin screen on the transmission side allows to efficiently increase the TL in high frequency, though lowering the mass-air-mass resonance, which leads to slightly reduced performance in

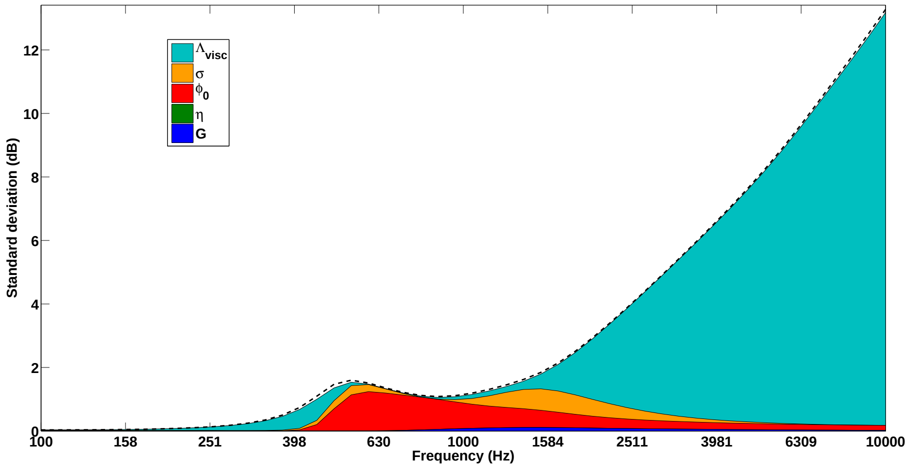

Figure 3. Sensitivity indices in the screen configuration

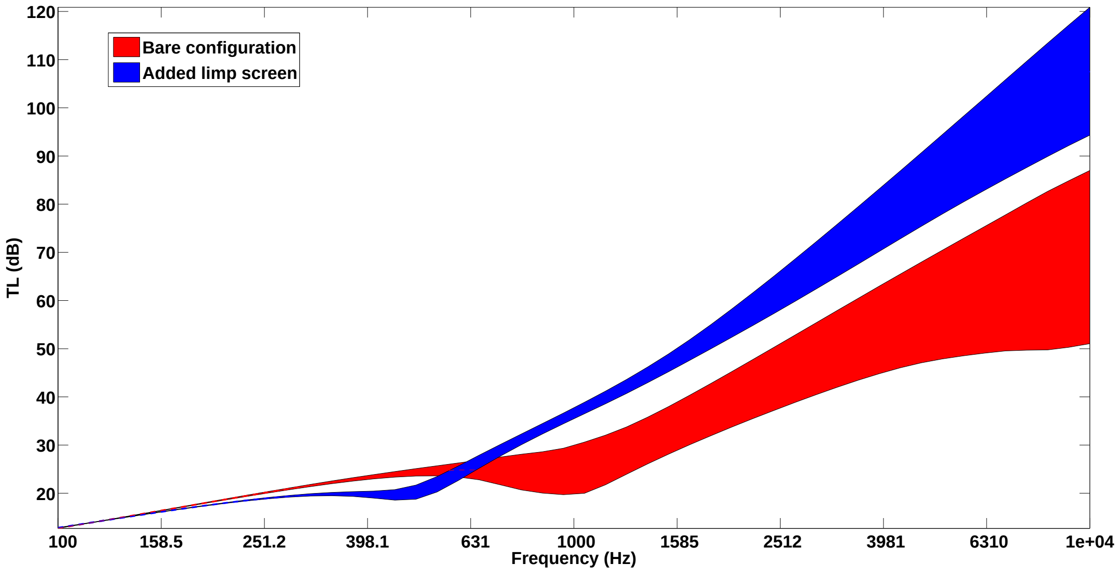

Figure 4: Transmission loss variability in the two configurations (average value ± standard deviation). Red: bare case ; blue: thin screen.

low frequency. In low frequency, the overall variability of the parameters stays low, well below 1dB, because none of the investigated parameters have an effect on the mass of the system, and the effect of poroelastic materials is usually rather weak in low frequencies.

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