
# WAVE BASED DESIGN OPTIMISATION OF COMPOSITE STRUCTURES OPERATING IN DYNAMIC ENVIRONMENTS

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## ABSTRACT

*The optimal mechanical and geometric characteristics for layered composite structures subject to vibroacoustic excitations are derived. A Finite Element description coupled to Periodic Structure Theory is employed for the considered layered panel. Structures of arbitrary anisotropy as well as geometric complexity can thus be modelled by the presented approach. Initially, a numerical continuum-discrete approach for computing the sensitivity of the acoustic wave characteristics propagating within the modelled periodic composite structure is exhibited. The first and second order sensitivities of the acoustic transmission coefficient expressed within a Statistical Energy Analysis context are subsequently derived as a function of the computed acoustic wave characteristics. Having formulated the gradient vector as well as the Hessian matrix, the optimal mechanical and geometric characteristics satisfying the considered mass, stiffness and vibroacoustic performance criteria are sought by employing Newton's optimisation method.*

## 1 INTRODUCTION

Layered and complex structures are nowadays widely used within the aerospace, automotive, construction and energy sectors with a general increase tendency, mainly because of their high stiffness-to-mass ratio and the fact that their mechanical characteristics can be designed to suit the particular purposes. Unluckily however, this high stiffness-to-mass ratio being responsible for the increased mechanical efficiency, at the same time induces high acoustic transmission through the structure. The need for simultaneously optimising an industrial structure of minimum mass and maximum static stiffness, while attaining satisfactory dynamic response performance levels is a challenging task for the modern engineer; especially when considering acoustic transmission through a layered structure which depends on the mechanical and geometric characteristics of each individual layer, resulting in a great number of design parameters to be optimised.

In this work an established wave based SEA approach is employed in order to predict the vibroacoustic performance of a composite layered panel. The novelty of the work focuses on the derivation of the first and second order sensitivity of the acoustic transmission coefficient expressed through SEA with respect to the structural design characteristics of the modelled structure. The considered design parameters include the entirety of the mechanical characteristics, the density as well as the thickness of each individual structural layer. Non conservative structural systems are also modelled by the exhibited approach. Employing a three dimensional FE description of the modelled structure allows for capturing the entirety of the sound transmitting propagating structural waves, while employing a PST formulation allows for drastically reducing the computational cost related to calculating the SEA parameters and the Hessian matrix for each configuration. Although not discussed in this work, the method is straightforward to apply to curved structures by expressing the FE structural matrices and wave propagation properties in polar coordinates.

## 2 ACOUSTIC WAVE SENSITIVITY

### 2.1 Formulation of the PST for an arbitrary structural segment

A periodic segment of a panel having arbitrary layering is hereby considered (see Fig.1) with $L_x$, $L_y$ its dimensions in the $x$ and $y$ directions respectively. The segment is modelled using a conventional FE software. The mass, damping and stiffness matrices of the segment $\mathbb{M}$, $\mathbb{C}$ and $\mathbb{K}$ are extracted and the DoF set $\mathbf{q}$ is reordered according to a predefined sequence such as:

$$\mathbf{q} = \{\mathbf{q_I} \;\; \mathbf{q_B} \;\; \mathbf{q_T} \;\; \mathbf{q_L} \;\; \mathbf{q_R} \;\; \mathbf{q_{LB}} \;\; \mathbf{q_{RB}} \;\; \mathbf{q_{LT}} \;\; \mathbf{q_{RT}}\}^\top \tag{1}$$

corresponding to the internal, the interface edge and the interface corner DoF (see Fig.1). The free harmonic vibration equation of motion for the modelled segment is written as:

$$[\mathbb{K} + \mathrm{i}\omega\mathbb{C} - \omega^{\mathbf{2}}\mathbb{M}]\mathbf{q} = \mathbf{0} \tag{2}$$

The analysis then follows as in [1] with the following relations being assumed for the displacement DoF under the passage of a time-harmonic wave:

$$\begin{aligned} \mathbf{q_R} =& \mathrm{e}^{-i\varepsilon_x}\mathbf{q_L}, \;\; \mathbf{q_T} = \mathrm{e}^{-i\varepsilon_y}\mathbf{q_B} \\ \mathbf{q_{RB}} =& \mathrm{e}^{-i\varepsilon_x}\mathbf{q_{LB}}, \;\; \mathbf{q_{LT}} = \mathrm{e}^{-i\varepsilon_y}\mathbf{q_{LB}}, \;\; \mathbf{q_{RT}} = \mathrm{e}^{-i\varepsilon_x - i\varepsilon_y}\mathbf{q_{LB}} \end{aligned} \tag{3}$$

with $\varepsilon_x$ and $\varepsilon_y$ the propagation constants in the $x$ and $y$ directions related to the phase difference between the sets of DoF. The wavenumbers $k_x$, $k_y$ are directly related to the propagation

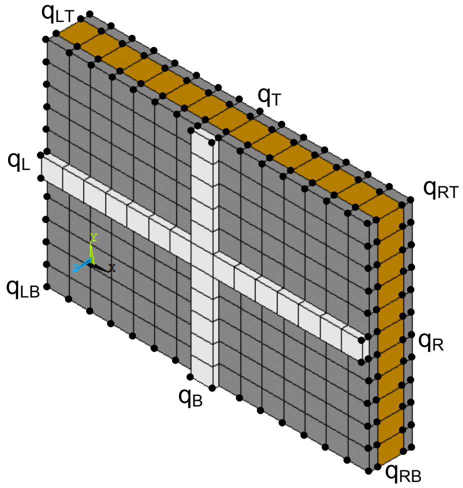


Figure 1. Caption of a FE modelled composite layered panel

constants through the relation:

$$\varepsilon_x = k_x L_x, \quad \varepsilon_y = k_y L_y \tag{4}$$

Considering Eq.3 in tensorial form gives:

$$\mathbf{q} = \begin{bmatrix} \mathbf{I} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I}\mathbf{e}^{-i\varepsilon_y} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I}\mathbf{e}^{-i\varepsilon_x} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\mathbf{e}^{-i\varepsilon_x} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\mathbf{e}^{-i\varepsilon_y} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{I}\mathbf{e}^{-i\varepsilon_x - i\varepsilon_y} \end{bmatrix} \mathbf{x} = \mathbf{R}\mathbf{x} \tag{5}$$

with $\mathbf{x}$ the reduced set of DoF: $\mathbf{x} = \{\mathbf{q_I} \;\; \mathbf{q_B} \;\; \mathbf{q_L} \;\; \mathbf{q_{LB}}\}^\top$. The equation of free harmonic vibration of the modelled segment can now be written as:

$$[\mathbf{R}^*\mathbb{K}\mathbf{R} + \mathrm{i}\omega\mathbf{R}^*\mathbb{C}\mathbf{R} - \omega^2\mathbf{R}^*\mathbb{M}\mathbf{R}]\mathbf{x} = \mathbf{0} \tag{6}$$

with $^*$ denoting the Hermitian transpose. The most practical procedure for extracting the wave propagation characteristics of the segment from Eq.6 is injecting a set of assumed propagation constants $\varepsilon_x$, $\varepsilon_y$. The set of these constants can be chosen in relation to the direction of propagation towards which the wavenumbers are to be sought and according to the desired resolution of the wavenumber curves. Eq.6 is then transformed into a standard eigenvalue problem and can be solved for the eigenvector $\mathbf{x}$ which describe the deformation of the segment under the passage of each wave type at an angular frequency equal to the square root of the corresponding eigenvalue $\lambda = \omega^2$. It is noted that the computed angular frequency quantities $\omega = \omega_r + \mathrm{i}\omega_i$ will have $\mid \omega_i \mid > 0$ implying complex values for the wavenumbers of the propagating wave types, otherwise interpreted as spatially decaying motion and from which the loss factor of each computed wave type $w$ can directly be determined.

A complete description of each passing wave including its $x$ and $y$ directional wavenumbers and its wave shape for a certain frequency is therefore acquired. It is noted that the periodicity condition is defined modulo $2\pi$, therefore solving Eq.6 with a set of $\varepsilon_x$, $\varepsilon_y$ varying from

0 to $2\pi$ will suffice for capturing the entirety of the structural waves. Further considerations on reducing the computational expense of the problem are discussed in [1]. It should be noted that only propagating waves will be considered in the subsequent analysis.

### 2.2 Parametric sensitivity

For an undamped structural segment the sensitivity of the real eigenvalues $\lambda_p$ can be written as

$$\frac{\partial \lambda_p}{\partial \beta_i} = \mathbf{x}_p^\top \left( \frac{\partial \mathbf{K}}{\partial \beta_i} - \lambda_p \frac{\partial \mathbf{M}}{\partial \beta_i} \right) \mathbf{x}_p \tag{7a}$$

$$\begin{aligned}\frac{\partial^2 \lambda_p}{\partial \beta_j \partial \beta_i} =& \mathbf{x}_p^\top \left( \frac{\partial^2 \mathbf{K}}{\partial \beta_j \partial \beta_i} - \lambda_p \frac{\partial^2 \mathbf{M}}{\partial \beta_j \partial \beta_i} - \frac{\partial \lambda_p}{\partial \beta_j}\frac{\partial \mathbf{M}}{\partial \beta_i} - \frac{\partial \lambda_p}{\partial \beta_i}\frac{\partial \mathbf{M}}{\partial \beta_j} \right) \mathbf{x}_p \\ & + \mathbf{x}_p^\top \left( \frac{\partial}{\partial \beta_j} \left[ \mathbf{K} - \lambda_p \mathbf{M} \right] \right) \frac{\partial \mathbf{x}_p}{\partial \beta_i} + \mathbf{x}_p^\top \left( \frac{\partial}{\partial \beta_i} \left[ \mathbf{K} - \lambda_p \mathbf{M} \right] \right) \frac{\partial \mathbf{x}_p}{\partial \beta_j}\end{aligned} \tag{7b}$$

with the sensitivity of the real mode shapes $\frac{\partial \mathbf{x}_p}{\partial \beta_j}$ to be calculated by the approach exhibited in [2]. The global mass and stiffness matrices $\mathbb{M}, \mathbb{K}$ of the structural segment are formed by adding the local mass and stiffness matrices of individual FEs. Eq.7 can be written as

$$\frac{\partial \lambda_p}{\partial \beta_i} = \mathbf{x}_p^\top \left( \mathbf{R}^* \frac{\partial \mathbb{K}}{\partial \beta_i} \mathbf{R} - \lambda_p \mathbf{R}^* \frac{\partial \mathbb{M}}{\partial \beta_i} \mathbf{R} \right) \mathbf{x}_p \tag{8a}$$

$$\begin{aligned}\frac{\partial^2 \lambda_p}{\partial \beta_j \partial \beta_i} =& \mathbf{x}_p^\top \left( \mathbf{R}^* \frac{\partial^2 \mathbb{K}}{\partial \beta_j \partial \beta_i} \mathbf{R} - \lambda_p \mathbf{R}^* \frac{\partial^2 \mathbb{M}}{\partial \beta_j \partial \beta_i} \mathbf{R} - \mathbf{R}^* \frac{\partial \lambda_p}{\partial \beta_j}\frac{\partial \mathbb{M}}{\partial \beta_i}\mathbf{R} - \mathbf{R}^* \frac{\partial \lambda_p}{\partial \beta_i}\frac{\partial \mathbb{M}}{\partial \beta_j}\mathbf{R} \right) \mathbf{x}_p + \\ & \mathbf{x}_p^\top \left( \frac{\partial}{\partial \beta_j} \left[ \mathbf{R}^* \mathbb{K} \mathbf{R} - \lambda_p \mathbf{R}^* \mathbb{M} \mathbf{R} \right] \right) \frac{\partial \mathbf{x}_p}{\partial \beta_i} + \mathbf{x}_p^\top \left( \frac{\partial}{\partial \beta_i} \left[ \mathbf{R}^* \mathbb{K} \mathbf{R} - \lambda_p \mathbf{R}^* \mathbb{M} \mathbf{R} \right] \right) \frac{\partial \mathbf{x}_p}{\partial \beta_j}\end{aligned} \tag{8b}$$

For the conservative system it is known however that $\frac{\partial \lambda_p}{\partial \beta_i} = \frac{\partial (\omega_p^2)}{\partial \beta_i}$, therefore

$$\frac{\partial \lambda_p}{\partial \beta_i} = \frac{\dfrac{\partial(\omega_p^2)}{\partial \omega_p}}{\dfrac{\partial \beta_i}{\partial \omega_p}} = 2\omega_p \frac{\partial \omega_p}{\partial \beta_i} \Leftrightarrow \frac{\partial \omega_p}{\partial \beta_i} = \frac{1}{2\omega_p}\frac{\partial \lambda_p}{\partial \beta_i} \tag{9a}$$

$$\frac{\partial^2 \lambda_p}{\partial \beta_j \partial \beta_i} = 2\frac{\partial \omega_p}{\partial \beta_j}\frac{\partial \omega_p}{\partial \beta_i} + 2\omega_p \frac{\partial^2 \omega_p}{\partial \beta_j \partial \beta_i} \Leftrightarrow \frac{\partial^2 \omega_p}{\partial \beta_j \partial \beta_i} = \frac{1}{2\omega_p}\left( \frac{\partial^2 \lambda_p}{\partial \beta_j \partial \beta_i} - 2\frac{\partial \omega_p}{\partial \beta_j}\frac{\partial \omega_p}{\partial \beta_i} \right) \tag{9b}$$

with $\omega_p$ the angular frequency at which the set of $\varepsilon_x$, $\varepsilon_y$ is true for the $p$ wave type described by the $\mathbf{x}_p$ deformation. For the wavenumber sensitivity $\frac{\partial k_p}{\partial \beta_i}$ the following is true

$$\frac{\partial k_p}{\partial \beta_i} = -\frac{\partial k_p}{\partial \omega_p}\frac{\partial \omega_p}{\partial \beta_i} = -\frac{1}{c_{g,p}}\frac{\partial \omega_p}{\partial \beta_i} \tag{10a}$$

$$\frac{\partial^2 k_p}{\partial \beta_j \partial \beta_i} = \frac{1}{c_{g,p}^3}\frac{\partial c_{g,p}}{\partial k_p}\frac{\partial \omega_p}{\partial \beta_j}\frac{\partial \omega_p}{\partial \beta_i} - \frac{1}{c_{g,p}}\frac{\partial^2 \omega_p}{\partial \beta_j \partial \beta_i} \tag{10b}$$

with $c_{g,p} = \frac{\partial \omega_p}{\partial k_p}$ the group velocity associated with the wave type $p$ at frequency $\omega_p$ and the quantities $c_{g,p}$, $\frac{\partial c_{g,p}}{\partial \omega_p}$ to be evaluated by the solution of the baseline structural design.

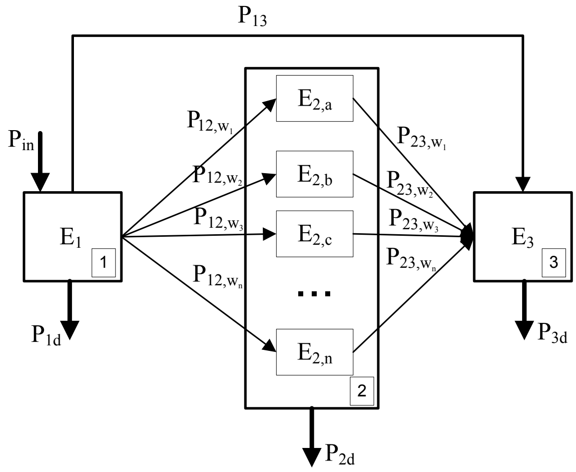


Figure 2: A schematic representation of the SEA power exchanges and energies for the modelled system.

## 3 SEA SENSITIVITY ANALYSIS

### 3.1 The employed SEA model

The impact of the parametric alterations on the vibroacoustic performance of the structure under investigation is exhibited in this section by deriving expressions for the sensitivity of the SEA results with respect to the propagating acoustic waves.

The total acoustic transmission coefficient $\tau$ is one of the most important indices of the vibroacoustic performance of a structure. The system to be modelled comprises one acoustically excited chamber (subsystem 1) and one acoustically receiving chamber (subsystem 3) separated by the modelled composite panel (subsystem 2). It is considered that each wave type is excited and transmits acoustic energy independently from the rest, therefore each considered wave type $w = w_1, w_2 ... w_n$ propagating within the composite panel is considered as a separate SEA subsystem. No flanking transmission is considered in the SEA model. The energy balance between the subsystems as it is considered within an SEA approach (see [3]) is illustrated in Fig.2, in which $E_1, E_3$ stand for the acoustic energy of the source room and the receiving room respectively and $E_2$ for the vibrational energy of the composite panel. Moreover $P_{in}$ is the injected power in the source room, $P_{1d}, P_{2d}$ and $P_{3d}$ stand for the power dissipated by each subsystem and $P_{13}$ is the non-resonant transmitted power between the rooms.

The derivation of an expression for the total acoustic transmission coefficient $\tau$ of the composite structure by merely accounting for its structural dynamic behaviour is exhibited in [4] and reads

$$\tau = \sum_{w=w_1}^{w_n} \tau_w + \frac{P_{13}}{P_{inc}} \tag{11}$$

with $\tau_w$ being the transmission coefficient of the wave type $w$ given as

$$\tau_w = \frac{8\rho^2 c^4 \pi \sigma_{rad,w}^2 n_w}{\rho_s \omega^2 A(\rho_s \omega \eta_w + 2\rho c \sigma_{rad,w})} \tag{12}$$

The non resonant transmission coefficient $\tau_{nr} = P_{13}/P_{inc}$ for a diffused acoustic field can be written as in [5]:

$$\tau_{nr}(\omega) = \frac{1}{\pi(\cos^2\theta_{min} - \cos^2\theta_{max})} \int_0^{2\pi} \int_0^{\theta_{max}} \frac{4Z_0^2}{\mid \mathrm{i}\omega\rho_s + 2Z_0 \mid^2} \sigma(\theta,\phi,\omega) \cos^2\theta \sin\theta \mathrm{d}\theta \mathrm{d}\phi \tag{13}$$

in which $\theta$ and $\phi$ are the incidence angle and the direction angle of the acoustic wave respectively and $Z_0 = \rho c/\cos\theta$ is the acoustic impedance of the medium. The term $\theta_{max}$ stands for the maximum incidence angle, accounting for the diffuseness of the incident field. It is hereby considered that $\theta_{max} = \pi/2$ for all the results presented in the current work. The term $\sigma(\theta,\phi,\omega)$ is the corrected radiation efficiency term. It is used in order to account for the finite dimensions of the panel and it is calculated using a spatial windowing correction technique presented in [6].

Eventually the STL of the panel can be expressed as

$$\mathrm{STL} = 10\log_{10}\left(\frac{1}{\tau}\right) \tag{14}$$

by definition.

### 3.2 Parametric sensitivity of the total acoustic transmission

In order to formulate the expression of the Hessian matrix describing the variation of the vibroacoustic performance of the structure with respect to its design parameters, the second order derivative of $\tau$ with respect to the considered set of parameters should be derived and expressed as

$$\frac{\partial\tau}{\partial\beta_i} = \sum_{w=w_1}^{w_n} \frac{\partial\tau_w}{\partial\beta_i} + \frac{\partial\tau_{nr}}{\partial\beta_i} \tag{15a}$$

$$\frac{\partial^2\tau}{\partial\beta_j\partial\beta_i} = \sum_{w=w_1}^{w_n} \frac{\partial^2\tau_w}{\partial\beta_j\partial\beta_i} + \frac{\partial^2\tau_{nr}}{\partial\beta_j\partial\beta_i} \tag{15b}$$

while the sensitivity of the STL index can directly be expressed with regard to $\tau$ as

$$\frac{\partial(STL)}{\partial\beta_i} = \frac{\mathrm{d}(STL)}{\mathrm{d}\tau}\frac{\partial\tau}{\partial\beta_i} = -\frac{10}{\ln(10)\tau}\frac{\partial\tau}{\partial\beta_i} \tag{16a}$$

$$\begin{aligned}\frac{\partial^2(STL)}{\partial\beta_j\partial\beta_i} &= \frac{\partial^2(STL)}{\partial\tau^2}\frac{\partial\tau}{\partial\beta_j}\frac{\partial\tau}{\partial\beta_i} + \frac{\partial(STL)}{\partial\tau}\frac{\partial^2\tau}{\partial\beta_j\partial\beta_i} \\ &= \frac{10}{\ln(10)\tau^2}\frac{\partial\tau}{\partial\beta_j}\frac{\partial\tau}{\partial\beta_i} - \frac{10}{\ln(10)\tau}\frac{\partial^2\tau}{\partial\beta_j\partial\beta_i}\end{aligned} \tag{16b}$$

In the following sections the evaluation of Eq.15 is discussed.

### 3.3 Modal density sensitivity

Using Courant's formula [7], the modal density of each wave type $w$ can be written at a propagation angle $\phi$ as a function of the propagating wavenumber and its corresponding group velocity $c_g$:

$$n_w(\omega,\phi) = \frac{Ak_w(\omega,\phi)}{2\pi^2|c_{g,w}(\omega,\phi)|} \tag{17}$$

The angularly averaged modal density of the structure is therefore given as

$$n_w(\omega) = \int_0^\pi n_w(\omega,\phi)\,\mathrm{d}\phi \tag{18}$$

Thanks to the chain differentiation rule the first and second order derivatives of the modal density for each wave type with respect to design variables $\beta_i$, $\beta_j$ can be expressed as

$$\begin{aligned}
\frac{\partial n_w(\omega,\phi)}{\partial \beta_i} &= \frac{\partial n_w(\omega,\phi)}{\partial k_w(\omega,\phi)}\frac{\partial k_w(\omega,\phi)}{\partial \beta_i} + \frac{\partial n_w(\omega,\phi)}{\partial c_{g,w}(\omega,\phi)}\frac{\partial c_{g,w}(\omega,\phi)}{\partial \beta_i} && \text{(19a)}\\
&= \frac{A}{2\pi^2|c_{g,w}(\omega,\phi)|}\frac{\partial k_w(\omega,\phi)}{\partial \beta_i} - \frac{Ak_w(\omega,\phi)\,\mathrm{sgn}(c_{g,w}(\omega,\phi))}{2\pi^2|c_{g,w}(\omega,\phi)|^2}\frac{\partial c_{g,w}(\omega,\phi)}{\partial k_w(\omega,\phi)}\frac{\partial k_w(\omega,\phi)}{\partial \beta_i}\\
\frac{\partial^2 n_w(\omega,\phi)}{\partial \beta_j\partial \beta_i} &= \frac{\partial^2 n_w(\omega,\phi)}{\partial k_w(\omega,\phi)^2}\frac{\partial k_w(\omega,\phi)}{\partial \beta_j}\frac{\partial k_w(\omega,\phi)}{\partial \beta_i} + \frac{\partial n_w(\omega,\phi)}{\partial k_w(\omega,\phi)}\frac{\partial^2 k_w(\omega,\phi)}{\partial \beta_j\partial \beta_i} && \text{(19b)}\\
&+ \frac{\partial^2 n_w(\omega,\phi)}{\partial c_{g,w}(\omega,\phi)^2}\frac{\partial c_{g,w}(\omega,\phi)}{\partial \beta_j}\frac{\partial c_{g,w}(\omega,\phi)}{\partial \beta_i} + \frac{\partial n_w(\omega,\phi)}{\partial c_{g,w}(\omega,\phi)}\frac{\partial^2 c_{g,w}(\omega,\phi)}{\partial \beta_j\partial \beta_i}\\
&= \frac{A}{2\pi^2|c_{g,w}(\omega,\phi)|}\frac{\partial^2 k_w(\omega,\phi)}{\partial \beta_j\partial \beta_i} + \frac{Ak_w(\omega,\phi)\,\mathrm{sgn}(c_{g,w}(\omega,\phi))}{\pi^2|c_{g,w}(\omega,\phi)|^3}\left(\frac{\partial c_{g,w}(\omega,\phi)}{\partial k_w(\omega,\phi)}\right)^2\frac{\partial k_w(\omega,\phi)}{\partial \beta_j}\frac{\partial k_w(\omega,\phi)}{\partial \beta_i}\\
&- \frac{Ak_w(\omega,\phi)\,\mathrm{sgn}(c_{g,w}(\omega,\phi))}{2\pi^2|c_{g,w}(\omega,\phi)|^2}\left(\frac{\partial^2 c_{g,w}(\omega,\phi)}{\partial k_w(\omega,\phi)^2}\frac{\partial k_w(\omega,\phi)}{\partial \beta_j}\frac{\partial k_w(\omega,\phi)}{\partial \beta_i} + \frac{\partial c_{g,w}(\omega,\phi)}{\partial k_w(\omega,\phi)}\frac{\partial^2 k_w(\omega,\phi)}{\partial \beta_j\partial \beta_i}\right)
\end{aligned}$$

while for the spatially averaged modal density

$$\frac{\partial n_w(\omega)}{\partial \beta_i} = \int_0^\pi \frac{\partial n_w(\omega,\phi)}{\partial \beta_i}\mathrm{d}\phi \tag{20a}$$

$$\frac{\partial^2 n_w(\omega)}{\partial \beta_j\partial \beta_i} = \int_0^\pi \frac{\partial^2 n_w(\omega,\phi)}{\partial \beta_j\partial \beta_i}\mathrm{d}\phi \tag{20b}$$

suggesting that the modal density sensitivity can be expressed merely by

- The sensitivity of the characteristics of the waves travelling within the considered structure with respect to the structural design (already determined in Sec.2).
- The sensitivity of the modal density with respect to the characteristics of the waves travelling in it.

A similar approach can be followed for computing all the remaining necessary SEA quantities.

### 3.4 Radiation efficiency sensitivity

In order to avoid the computationally inefficient frequency and directional averaging of the modal dependent radiation efficiency sensitivity $\frac{\partial \sigma_{rad,w}(\omega,\phi)}{\partial \beta_i}$, it is practical to employ expressions introducing a direct wavenumber dependence of $\sigma_{rad,w}$ such as the ones exhibited in [1, 8, 9]. For a generic periodic structure including discontinuities the assumption of sinusoidal mode shapes is no longer valid, therefore the radiation efficiency should be calculated directly from the PST derived wave mode shapes. The radiation efficiency expression as derived in [1] can therefore be employed. For continuous structures, mode shapes of sinusoidal form can be assumed in order to avoid any FE discretization errors in the solution. The set of asymptotic formulas given in [9] can be used for computing the averaged wavenumber dependent radiation efficiency of the panel as

$$\sigma_{rad,w} = \frac{1}{\sqrt{1-\mu^2}} \qquad \mu < 0.90 \tag{21a}$$

$$\sigma_{rad,w} = \frac{L_x + L_y}{\pi\mu\kappa L_x L_y \sqrt{\mu^2 - 1}} \left( \ln\left(\frac{\mu+1}{\mu-1}\right) + \frac{2\mu}{\mu^2 - 1} \right) \qquad \mu > 1.05 \tag{21b}$$

$$\sigma_{rad,w} = (0.5 - 0.15 \min(L_x, L_y)/\max(L_x, L_y)) \sqrt{k \min(L_x, L_y)} \qquad \mu = 1 \tag{21c}$$

with $\mu = \left(\frac{k_x^2 + k_y^2}{\kappa^2}\right)^{1/2}$, where $\kappa = \omega/c$ is the acoustic wavenumber. In the region $0.90 < \mu < 1.05$ a shape preserving Hermite interpolation function is employed assuring the continuity and double differentiability for the entire spectrum of the $\sigma_{rad,w}$ expression. The sensitivity expressions for the radiation efficiency of the panel can therefore be derived as a function of the propagating flexural wavenumbers by Eq.21, while the interpolation function is used for expressing the sensitivity of $\sigma_{rad,w}$ for the remaining spectrum.

## 4 NUMERICAL CASE STUDIES

In order to validate the exhibited optimisation approach, an asymmetric sandwich panel comprising two facesheets and a core is modelled in this section. The lower facesheet has a thickness $h_1$=1mm and is made of a material having $\rho_{m,1}$=3000e$^{-9}$kg/mm$^3$, $E_1 = 70$GPa and a Poisson's ration $v_1$=0.1. The upper facesheet has a thickness equal to $h_3$=2mm and is made of the same material as the lower facesheet. The core has a thickness $h_2$=10mm and is made of a material with $\rho_{m,2}$=50e$^{-9}$kg/mm$^3$, $E_2 = 0.07$GPa and $v_2$=0.4. Three FEs are used in the sense of thickness in order to model the structure. All computations were conducted using the R2013a version of MATLAB®.

### 4.1 Structural design optimisation of the layered structure

As discussed in Sec.2, the criteria to be considered within the optimisation process of the mechanical and geometric characteristics of the panel are its mass, stiffness and vibroacoustic performance. The surface mass of the panel $\rho_s$ is chosen as a representative mass index, the total acoustic transmission coefficient $\tau$ is selected as the vibroacoustic performance index, while with regard to the structural stiffness and for the sake of simplicity we will hereby assume that we are solely interested in the sum of the static flexural stiffnesses of the panel $D_{xx}$, $D_{yy}$ expressed in the case of an isotropic composite panel as

$$d_s = \frac{2}{3} \sum_{l=l_1}^{l_{max}} \left( Q_l (z_l^3 - z_{l-1}^3) \right) \tag{22}$$

with $z_l$ the coordinate of the upper surface of layer $l$ in the thickness direction. The design cost functions, employed in order to decide the relation between $\rho_s$, $\tau$ and $d_s$ and the corresponding induced design cost are exhibited in Fig.3.

Additional constraints (e.g. minimum axial and/or flexural stiffness, maximum surface mass e.t.c) can be considered. The constrained optimization problem is solved using Newton's method.

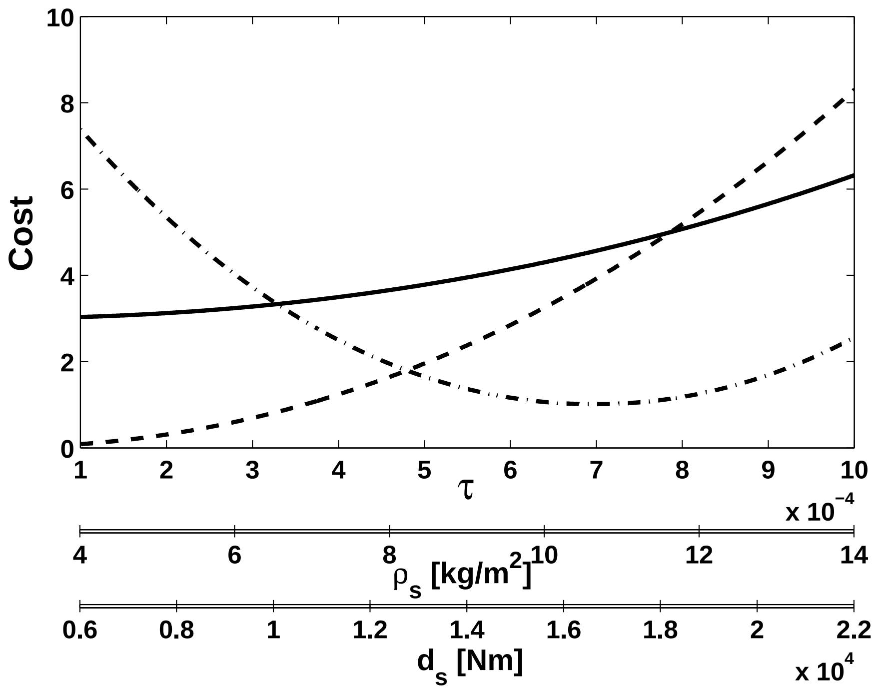


Figure 3: Representation of the cost functions employed within the current optimisation process. Cost function corresponding to: The acoustic transmission coefficient $\tau$ (−), The surface mass density $\rho_s$ (−−), The flexural stiffness $d_s$ of the panel (− · −)

### 4.2 Optimal parameters and discussion on the computational efficiency

The optimisation problem is solved for $k = 0.13$rad/mm, and the optimal material and geometric parameters that minimise the sum of the costs as presented in Fig.3 are computed as follows

$$E_1 = 80.9\text{GPa},\ v_1 = 0.12,\ h_1 = 1.19\text{mm},\ \rho_{m,1} = 1647\text{kg/m}^3$$
$$E_2 = 110\text{MPa},\ v_2 = 0.37,\ h_2 = 10.53\text{mm},\ \rho_{m,2} = 14.6\text{kg/m}^3$$
$$E_3 = 58.3\text{GPa},\ v_3 = 0.19,\ h_3 = 1.74\text{mm},\ \rho_{m,3} = 1500\text{kg/m}^3$$

It is noted that the only quantities laying on the limits of the predefined constraints which could potentially further improve the overall structural performance are the Young's modulus of the core layer $E_2$ as well as the mass density of the upper layer $\rho_{m,3}$. Optimising the structure in a broadband frequency range can be done by averaging the optimal parameters over the frequency range of interest or by introducing a weighting average for the frequency bands that are considered more important (e.g. frequency of the external acoustic excitation). The optimisation process was completed in 8 iterations each of which lasted approximately 78 seconds, resulting in a total computation time of 630s. This suggests that a broadband structural optimisation is feasible within a few hours, even with a conventional computing equipment.

## 5 CONCLUSIONS

In this work, the optimal mechanical and geometric characteristics for layered composite structures subject to vibroacoustic excitations were derived in a wave SEA context. The main conclusions of the paper are summarised as:

(i) An intense frequency dependent variation of the sensitivity of the propagating wave characteristics has been observed as a function of the design of the composite structure. This also implies frequency dependence of the optimal design parameters.

(ii) Expressions for the first and second order sensitivities of the SEA quantities, namely the modal density and the radiation efficiency of the composite panel were derived. The design parametric sensitivity for each of the SEA quantities, as well as of the acoustic transmission coefficient were found to be highly frequency dependent. The impact of the design alteration on the vibroacoustic response was maximised in the vicinity of the acoustic coincidence range for most parameters.

(iii) The suggested optimisation process is computationally efficient, allowing for a broadband structural optimisation of a layered structure in a rational period of time, even with the use of a conventional computing equipment.

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