THE SOUND TRANSMISSION LOSS USING THE STOCHASTIC WAVE FINITE ELEMENT METHOD

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ABSTRACT

Anisotropic and sandwich structures are used in many engineering areas such as aerospace and automotive constructions. These types of structures are often used because of their high stiffness to mass ratios. However these structures oftentimes present a compromise between their mechanical and vibro-acoustic behaviour. The vibro-acoustic study for the anisotropic and sandwich structures is well developed during the last years.

There are many methods which allow the computation of the wavenumbers for isotropic and anisotropic structures. Analytical formulas exist to calculate the wavenumbers of anisotropic plates based on the Classical Laminate Plate Theory. To take into account shear deformation, Whitney suggested the formulation of the First-order Shear Deformation Theory (FSDT). A model for an infinite sandwich panel by including the description of symmetric and antisymmetric motions was developed. Leppington expressed the radiation efficiency of a rectangular panel as well as the vibroacoustic response under a reverberant field of thin orthotropic panels.

To deal with the wave characteristics in periodic structures, the Wave Finite Element Method (WFEM) is used. This spectral formulation is a result of a coupling between the conventional finite element method and the periodic structure theory. Its formulation starts with the discretization of the studies structure. An eigenvalue problem is then formulated using the periodicity of the structure. The general theory of the WFE is proposed by Mead and was improved by Zhong and Williams. This approach is then used for predicting the acoustic behavior of anisotropic plates. It investigates the evolution of radiation efficiency and sound transmission loss with frequency.

In all presented formulations, the input parameters are deterministic. However for layered structures, there is a high variability of mechanical parameters. The main novelty of this paper is investigating the effects of the uncertain mechanical parameters on the acoustic behaviour of anisotropic structures, especially in mid- and high frequencies.

This paper discusses the effect of uncertain parameters on vibro-acoustic behavior, especially on the Sound Transmission Loss (STL) of composite panels. The formulation presented is hybridization between spectral, energetic and uncertain methods. The Uncertain inputs parameters are represented using a parametric probabilistic approach which allows for the separation between the deterministic and the stochastic components in the process.

The second order stochastic parameters are developed using the generalized polynomial chaos expansion. In order to evaluate the outputs, there are two different methods: intrusive and non-intrusive methods. The efficiency of the approach is exhibited for isotropic panels.

1 INTRODUCTION

Anisotropic and sandwich structures are used in many engineering areas such as aerospace and automotive constructions. To deal with this type of structures in high frequencies, the Statistical Energy Analysis (SEA) is often used to predict the dynamic behavior of structures. The SEA method is based on the calculation of the energy quantities exchanged between the sub-systems. In the case of structural wave modelling, waves represent SEA subsystems, and the use of the SEA consist on evaluating the energy exchange between waves. Before using an SEA approach, the identification of the propagating waves is first investigated to obtain the spectrum of the wave dispersion characteristics. There are many methods which allow the computation of the wavenumbers for isotropic and anisotropic structures. Analytical formulas exist for the calculation of wavenumbers of anisotropic plates based on the Classical Laminate Plate Theory [1]. To take into account shear deformation, Whitney *et al.* [2] suggested the formulation of the First-order Shear Deformation Theory (FSDT). Dym and Lang [3] developed a model for an infinite sandwich panel by including the description of symmetric and antisymmetric motions. A Higher-order Shear Deformation Theory (HSDT), initially conceived in [4] is applied in [5] for expressing the vibroacoustic response of a structure within an SEA context. Leppington *et al.*[6] expressed the radiation efficiency of a rectangular panel as well as the vibroacoustic response under a reverberant field [7] of thin orthotropic panels.

This paper discusses the effect of uncertain parameters on vibro-acoustic behavior of composite panels. The formulation presented is a hybridization between spectral, energetic and uncertain methods. The Uncertain inputs parameters are represented using a parametric probabilistic approach which allows for the separation between the deterministic and the stochastic components in the process. The second order stochastic parameters are developed using the generalized polynomial chaos expansion. In order to evaluate the outputs, there are two different methods: intrusive and non-intrusive methods. The first one consists in projecting the process using a Galerkin approach to obtain a set of deterministic equations instead of the stochastic one. The second method is based on simulations of the deterministic model before an adequate post-processing to evaluate the uncertainty of the output parameters. In this paper, different methods are presented and discussed.

2 WAVE BASED PREDICTION OF THE VIBROACOUSTIC PERFORMANCE FOR A COMPOSITE STRUCTURE

2.1 Wave propagation analysis by a 2D Finite Element method

A rectangular periodic composite panel composed by N identical sub-structures is considered. The dimensions of the panel are : L_x, L_y and h its thickness(see fig.1). Using the conventional finite element method, a single periodic segment of the composite panel is modeled and the mass and stiffness matrices are extracted

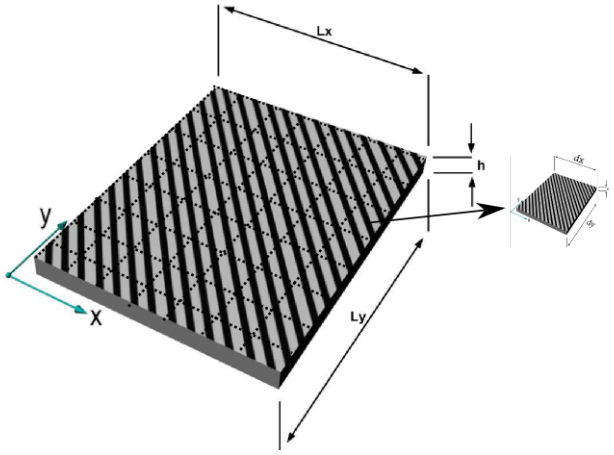


Figure 1. A periodic composite panel

The entries for each Degree of Freedom (DoF), of the nodes laying on the same edge of the segment, say edges Q, R, S and T, are organized in the mass and stiffness matrices so that the displacements can be written as: $\mathbf{u} = \{\mathbf{u_Q}\ \mathbf{u_R}\ \mathbf{u_S}\ \mathbf{u_T}\}^T$. Following the analysis presented in [8] the time-harmonic equation of motion of the segment assuming uniform and structural damping can be written as:

$$\left(\mathbf{K}\left(1+\eta \mathrm{i}\right)-\omega^2\mathbf{M}\right)\mathbf{u}=\mathbf{F} \tag{1}$$

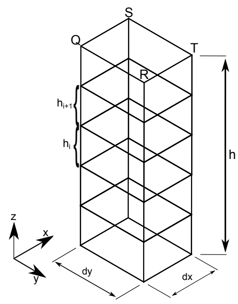


Figure 2. View of the modeled periodic segment with its edges Q, R, S and T

where η is the structural damping coefficient, ω is the angular frequency and **F** the vector of the nodal forces. The dynamic stiffness matrix can be written as :

$$\mathbf{D}=\mathbf{K}\left(1+\eta \mathrm{i}\right)-\omega^2\mathbf{M} \tag{2}$$

therefore equation (1) may be written as:

$$\begin{bmatrix} \mathbf{D_{QQ}} & \mathbf{D_{QR}} & \mathbf{D_{QS}} & \mathbf{D_{QT}} \\ \mathbf{D_{RQ}} & \mathbf{D_{RR}} & \mathbf{D_{RS}} & \mathbf{D_{RT}} \\ \mathbf{D_{SQ}} & \mathbf{D_{SR}} & \mathbf{D_{SS}} & \mathbf{D_{ST}} \\ \mathbf{D_{TQ}} & \mathbf{D_{TR}} & \mathbf{D_{TS}} & \mathbf{D_{TT}} \end{bmatrix} \begin{Bmatrix} \mathbf{u_Q} \\ \mathbf{u_R} \\ \mathbf{u_S} \\ \mathbf{u_T} \end{Bmatrix} = \begin{Bmatrix} \mathbf{F_Q} \\ \mathbf{F_R} \\ \mathbf{F_S} \\ \mathbf{F_T} \end{Bmatrix} \tag{3}$$

Using periodic structure theory for the modelled segment and assuming a time-harmonic response the displacements of each edge can be written as a function of the displacements at one single edge. Taking edge Q as the edge of reference we have:

$$\mathbf{u_R}=\lambda_x\mathbf{u_Q},\ \mathbf{u_S}=\lambda_y\mathbf{u_Q},\ \mathbf{u_T}=\lambda_x\lambda_y\mathbf{u_Q} \tag{4}$$

Using the same theory, the force vectors can be written as:

$$\mathbf{F_R} = \lambda_x \mathbf{F_Q},\ \mathbf{F_S} = \lambda_y \mathbf{F_Q},\ \mathbf{F_T} = \lambda_x \lambda_y \mathbf{F_Q} \tag{5}$$

With λ_x and λ_y the phase constants which are related to the wavenumbers k_x and k_y through the relation:

$$\lambda_x = \mathrm{e}^{-ik_x d_x},\ \lambda_y = \mathrm{e}^{-ikyd_y} \tag{6}$$

The displacement vector can therefore be written as:

$$\begin{Bmatrix} \mathbf{u_Q} \\ \mathbf{u_R} \\ \mathbf{u_S} \\ \mathbf{u_T} \end{Bmatrix} = \begin{Bmatrix} \mathbf{I} \\ \lambda_x \mathbf{I} \\ \lambda_y \mathbf{I} \\ \lambda_x \lambda_y \mathbf{I} \end{Bmatrix} \mathbf{u_Q} \tag{7}$$

Assuming no external excitation, the equilibrium conditions along edge Q implies that:

$$\left\{ \mathbf{I} \quad \lambda_y^{-1}\mathbf{I} \quad \lambda_x^{-1}\mathbf{I} \quad \lambda_x^{-1}\lambda_y^{-1}\mathbf{I} \right\} \begin{Bmatrix} \mathbf{F_Q} \\ \mathbf{F_R} \\ \mathbf{F_S} \\ \mathbf{F_T} \end{Bmatrix} = 0 \tag{8}$$

Eventually, substituting equation (7), (8) in equation (1) we end up with the eigenproblem:

$$\left\{ \mathbf{I} \quad \lambda_y^{-1}\mathbf{I} \quad \lambda_x^{-1}\mathbf{I} \quad \lambda_x^{-1}\lambda_y^{-1}\mathbf{I} \right\} \mathbf{D} \begin{Bmatrix} \mathbf{I} \\ \lambda_x \mathbf{I} \\ \lambda_y \mathbf{I} \\ \lambda_x \lambda_y \mathbf{I} \end{Bmatrix} \mathbf{u_Q} = 0 \tag{9}$$

2.2 Calculation of the modal density

Using the Courant's formula [9], the modal density of each propagating wave type w can be written for each angle ϕ as a function of the propagating wavenumber (obtained by the WFE 2D 2.1) and its corresponding group velocity c_g:

$$n_w(\omega, \phi) = \frac{A\, k_w(\omega, \phi)}{2\pi^2\, |c_{g,w}(\omega, \phi)|} \tag{10}$$

where A is the area of the panel and the group velocity is expressed as:

$$c_g(\omega, \phi) = \frac{\mathrm{d}\omega}{\mathrm{d}k(\omega, \phi)} \tag{11}$$

The averaged modal density of the structure is eventually given as:

$$n_w(\omega) = \int_0^\pi n_w(\omega, \phi)\, \mathrm{d}\phi \tag{12}$$

2.3 Calculation of the radiation efficiency

In order to calculate the radiation efficiency $\sigma\left(k\left(\omega\right)\right)$ for each propagating wave type, the set of asymptotic formulas given in [6] can be used in order to compute $\sigma\left(k\left(\omega\right)\right)$. Within an SEA context, energy equipartition amongst the resonant modes is assumed so that the radiation efficiency is expressed as:

$$\sigma_{rad}\left(\omega\right) = \frac{1}{n\left(\omega\right)}\int_0^{\pi} \sigma\left(k\left(\omega,\phi\right)\right) n\left(\omega,\phi\right) d\phi \tag{13}$$

For a periodic discontinuous structure assuming sinusoidal mode shapes is no longer valid; therefore the radiation efficiency should be computed directly from the WFEM derived wave mode shapes. The radiation efficiency expression given in [10] can be employed for this purpose.

3 EMPLOYING THE GENERALIZED POLYNOMIAL CHAOS EXPANSION (GPCE) WITHIN THE VIBROACOUSTIC RESPONSE MODELLING

The polynomial chaos expansion is an efficient tool for describing uncertainty propagation in mechanical systems. It consist on separating between the stochastic components of a random function and its deterministic components. This theory, developed by Wiener [11], helps to expand any second order process u (with finite variance) in a series of orthogonal polynomials as:

$$u = u_0 H_0 + \sum_{i_1=1}^{\infty} u_{i_1} H_1(\xi_{i_1}) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1} u_{i_1 i_2} H_2(\xi_{i_1},\xi_{i_2}) + \sum_{i_1=1}^{\infty}\sum_{i_2=1}^{i_1}\sum_{i_3=1}^{i_2} u_{i_1 i_2 i_2} H_3(\xi_{i_1},\xi_{i_2},\xi_{i_3}) + \cdots \tag{14}$$

where $H_p(\xi_{i_1},\cdots,\xi_{i_p})$ represents orthogonal polynomial (chaos polynomials) of order p. In practice, the polynomial chaos expansion is truncated to a finite number of terms. In a compact form, equation (14) can be expressed as:

$$u \approx \sum_{i=0}^{P} u_i \Psi_i(\xi) \quad , \quad G = \sum_{k=0}^{p} C_{M+k-1}^{k} = \frac{(M+p)!}{M!p!} \tag{15}$$

where $\xi = [\xi_{i_1},\cdots,\xi_{i_p}]^T$, and M denoting the number of the uncertain parameters.
Since in most applications the stochastic input variables are not normal, Xiu and Karniadakis [12] proposed a generalized form of Hermite polynomial chaos expansion using other orthogonal polynomials in terms of non-Gaussian random variables called wiener-askey. Table 1 resumes usual random variables and their orthogonal polynomials.

	Random variable ξ	Winer-Askey chaos $\Psi(\xi)$	Support
Continue distributions	Gaussian	Hermite	$(-\infty,+\infty)$
	Uniform	Legendre	[a,b]
	Gamma	Laguerre	$[0,\infty]$
	Beta	Jacobi	$[a,b]$
Discrete distribution	Poisson	Charlier	$\{0,1,\cdots,\}$
	binomial	Krawtchouk	$\{0,1,\cdots,N\}$

Table 1: Correspondence between the choice of polynomial and given distribution of usual random variables

When the input parameters have not a non-Gaussian behavior, the parametrization of the problem is quite difficult. Rosenblatt [13] proposed a simple transformation of non-Gaussian distributions to Gaussian ones. Some analytical transformations are mentioned in the following table:

Distribution	Transformation
Uniform (a, b)	$a + (b - a)\left(0.5 + 0.5erf(\xi/\sqrt{2})\right)$
Normal (μ, σ)	$\mu + \sigma\xi$
Lognormal (μ, σ)	$exp(\mu + \sigma\xi)$
Gamma (a, b)	$ab\left(\xi\sqrt{(}\frac{1}{9a} + 1 - \frac{1}{9a})\right)^3$
Exponential (λ)	$-\frac{1}{\lambda}log\left(\frac{1}{2} + \frac{1}{2}erf(\xi/\sqrt{2})\right)$

Table 2. Random variables and their transformations

with $erf(x) = \frac{2}{\sqrt{\Pi}}\int_0^x e^{-t^2}dt$.

4 NUMERICAL VALIDATIONS

This section deals with numerical validations of the proposed formulation. As presented above, the formulation is a hybridization of an energy based approach, the wave finite element method and a parametric probabilistic approach. The objective of the approach is identifying the effects of uncertain parameters on the acoustic and vibro-acoustic behaviour of panels.

4.1 Isotropic honeycomb panel

In order to validate the suggested models, the first case study is evaluating the acoustic parameters for an isotropic honeycomb panel. The mechanical properties of facesheets and the core are mentioned in Table 3:

	E (Pa)	ρ (kg/m^3)	thickness (m)	Poisson's ratio	Structural damping
Facesheets	$49 \cdot 10^9$	1600	$5 \cdot 10^{-4}$	0.15	1 %
Core	$1.951 \cdot 10^8$	160	$6.3 \cdot 10^{-3}$	0.15	1 %

Table 3. Mechanical characteristics of facesheets and the core for the isotropic panel

The structure dimensions are : $L_x = 0.84$ m and $L_y = 0.42$ m. Regarding the periodicity of the panel, only one periodic segment with $d_x = 0.02$ (m) and $d_y = 0.005$ (m) is discretized using the conventional finite element method. The mass and stiffness matrices are then extracted in order to formulate the polynomial eigenvalue problem exhibited above. Knowing that the structure is an isotropic one, suggests that the wave properties are the same in all propagation directions in the structure. Therefore solving the eigenproblem for only one direction of propagation will suffice for capturing the entirety of the wave propagation data for the panel.

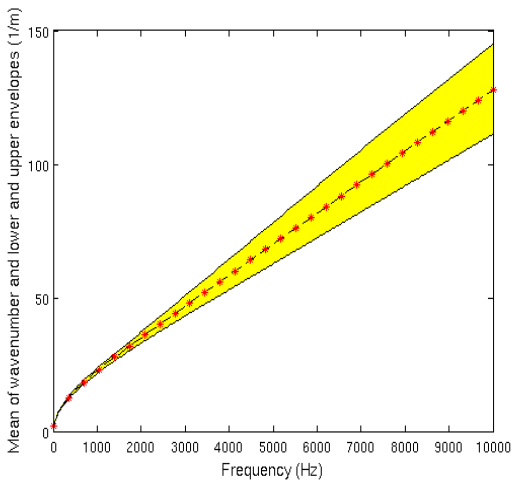
In order to apply the stochastic process, the mechanical parameters are assumed to be uncertain with different evolution. Table 4 summarizes the different stochastic parameters and their distributions. The choice of the Lognormal distribution is used regarding the positivity of the uncertain parameters.

Random variables	Type of distribution	Mean	Standard deviation
Young modulus of facesheets (Pa)	Lognormal	$49 \cdot 10^9$	5%
Density of facesheets (kg m^{-3})	Lognormal	1600	5%
Young modulus of core (Pa)	Lognormal	$1.951 \cdot 10^8$	10%
Density of core (kg m^{-3})	Lognormal	160	10%
Damping	Uniform	0.01	5%

Table 4. Random variables

In this stochastic calculations step, the isoprobabilistic transformations are used to move from a non-Gaussian distribution to a Gaussian one. Then, the Latin Hypercube Sampling is performed to apply the stochastic process with lower computation effort.

The wavenumber values for the first flexural wave of the isotropic sandwich structure are presented in Fig.3. In the same figure the envelope representing the min-max wavenumber due to the input stochastic parameters, as well as the standard deviation of the wavenumber values are also exhibited. It should be noted that the out of plane structural motion of the flexural wave is responsible for transmitting the vast majority of acoustic energy, therefore this will be the main wave type taken into account during the subsequent analysis. It is observed that the effect of parametric uncertainties on the flexural wavenumber is small for low frequencies ($<$ 1000 Hz) with a maximum deviation of approximately 1.5%. With an increasing frequency the effect of the structural parametric uncertainties on the wavenumber becomes more evident, with the maximum deviation from the mean value being equal to 13.4% at the highest frequency of the analysis (10 kHz). Considering the standard deviation of the flexural wavenumber values a piece-wise linearity is observed. The first low frequency region is observed up to frequencies of 1000 Hz while for higher frequencies a second linear region of a higher gradient is exhibited. With regard to both the results of the wavenumber as well as its standard deviation values an excellent agreement is observed between the presented approach and the Monte Carlo simulation results. It is noted that 4000 samples were considered during the Monte Carlo simulation.



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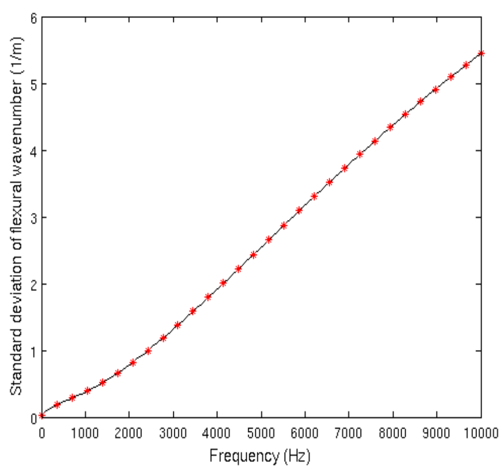

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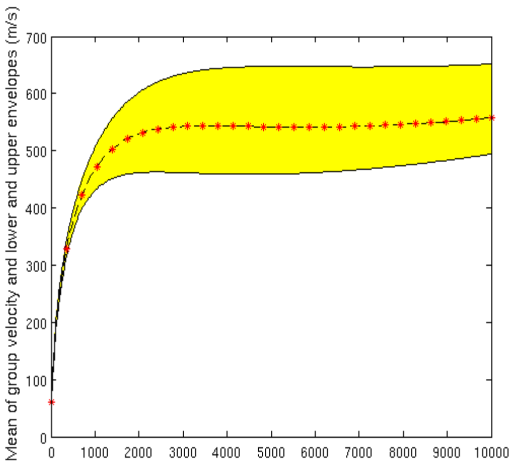
Figure 3: Wavenumber : (a) mean and min-max envelop, (b) Standard deviation, (-) WFE-Chaos, (*) Monte Carlo



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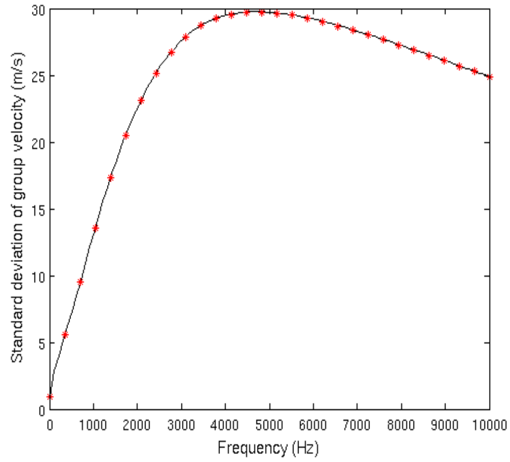

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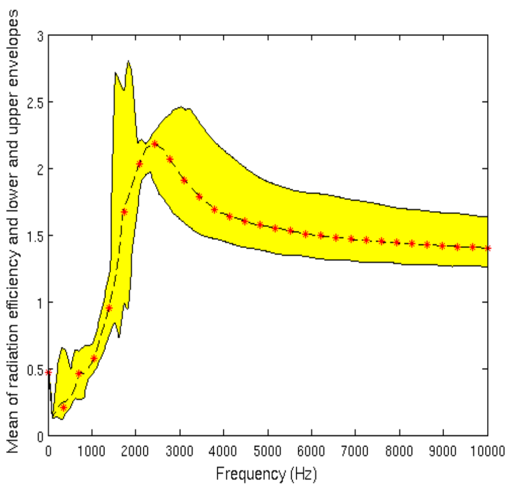
Figure 4: Group velocity : (a) mean and min-max envelop, (b) Standard deviation, (-) WFE-Chaos, (*) Monte Carlo

The group velocity results for the first flexural wave of the isotropic sandwich structure are presented in Fig.4. In the same figure the envelope representing the min-max group velocity values due to the input stochastic parameters, as well as the standard deviation of the group velocity are also presented. As with the wavenumber results it can be observed that for low frequencies (< 800 Hz) the impact of parametric uncertainties on the group velocity values of the flexural wavenumber is insignificant. For higher frequencies the effect of the structural parametric uncertainties on the group velocity results becomes important, with the maximum deviation from the mean value being equal to 19.2% at 5 kHz. With regard to the standard deviation of the flexural wavenumber group velocity it can be observed that it increases up to a certain frequency where it attains a maximum value; that is at approximately 5000 Hz. Again, as with the wavenumber results an excellent agreement is observed between the exhibited approach and the Monte Carlo simulation results.



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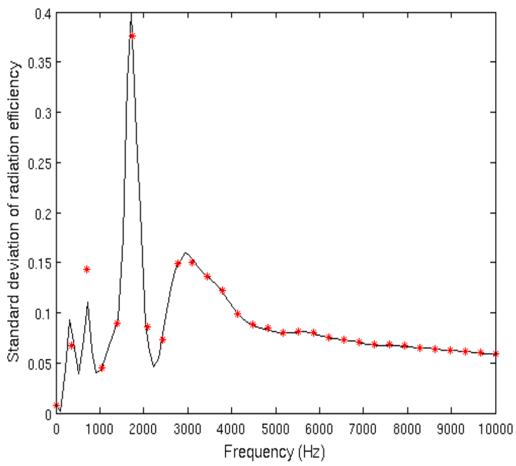

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Figure 5: Radiation efficiency :: (a) mean and min-max envelop, (b) Standard deviation, (-) WFE-Chaos, (*) Monte Carlo

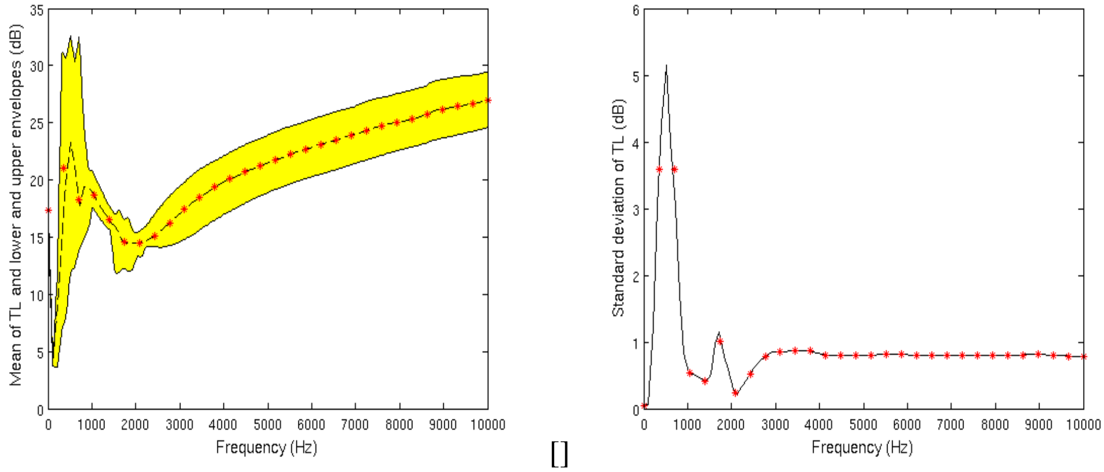

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Figure 6: Transmission Loss: (a) mean and min-max envelop, (b) Standard deviation, (-) WFE-Chaos, (*) Monte Carlo

5 CONCLUSIONS

The modelling of the vibro-acoustic behaviour of composite layered structures with uncertain parameters was considered in this paper. The presented approach is a combination of a wave based SEA approach and a parametric probabilistic approach. The first method consists in evaluating the wave propagation characteristics within composite structures. A spectral method, based on the periodicity of the structure studied is presented. Then, the SEA can be applied to identify the evolution of energy quantities between different sub-structures. In our case, all waves are considered as substructures. This approach leads to obtain vibro-acoustic indices such as the radiation efficiency and the sound transmission loss for each considered wave type.

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