MODELING AND IDENTIFICATION OF A CLASS OF HYPERVISCOELASTIC MATERIAL BEHAVIOR

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ABSTRACT

Elastomers are widely used in aerospace, automotive and civil engineering applications thanks to their ability to undergoing high strains in large temperature range. To study the nonlinearities, several models have been developed in the literature. In this work, a nonlinear viscoelastic model at finite strain is developed based upon functional and internal variable approaches and the time strain superposition principle (TSSP). Constitutive equations for the stress are derived such that the second law of thermodynamic, in the form of Clausius-Duhem inequality, is satisfied. Identification of several model' parameters is studied using experimental data of pure shear, simple extension tests on a natural rubber NR and a least square minimization procedure.

Keywords: *nonlinear viscoelasticity, finite strain, time-strain superposition principle, identification, discretization of constitutive equations.*

1 INTRODUCTION

Viscoelastic materials, such as elastomers, are well known by their nonlinear time dependent behaviour and large deformation. In the literature, two main approaches are used to determine constitutive equations for these materials: functional approach and internal variables approach.

The first one consists on expressing the free energy density as a multiple integral functional in terms of the history of deformation (Christensen [1] Pipkin [2]), and then the stress tensor is derived from this functional with respect to the strain tensor, taking into account the whole thermodynamic assumptions. Whereas, the second one consists on expressing the free energy density as a Taylor series in terms of the strain tensor and a set of internal variables (second order tensor akin to the second Piola-Kirchhoff stress tensor Simo [3] Govindjee [5] or the right Cauchy-Green strain tensor Holzapfel [4] Valanis [6] Schapery [7]). The viscoelastic behavior is described with a linear rate equation governing the set of internal variables and the thermodynamic restrictions are expressed by the Clausius-Duhem inequality.

In this study, a model is developed for nonlinear viscoelastic materials undergoing large mechanical process. As a first step, a rheological motivation is investigated. Then, the nonlinear viscoelastic model is introduced using a formulation based upon both functional and internal variables approaches and the time strain superposition principle TSSP (reduced time). Constitutive equations for stress and dissipation are then derived from the thermodynamic principles. Finally, an identification procedure of several model parameters is highlighted using pure shear and simple extension tests on a natural rubber material (NR).

2 NONLINEAR VISCOELASTIC MODEL

2.1 Rheological model

The use of rheological models to describe the mechanical behavior of several materials is very widespread. The advantage of such model is its simplicity to obtain the constitutive equations of a material. It consists on associating elementary rheological models, namely a Hookean element (spring) and a Newtonian fluid element (dashpot). This association could be in series and/or in parallel.

The aim of this section is to develop the viscoelastic model proposed by Simo [3] using a rheological model of Zener. For that, we shall investigate some assumptions in the definition of internal variables. The viscoelastic model is developed using the concept of internal variables; the stress is used as an internal variables and denoted Q.

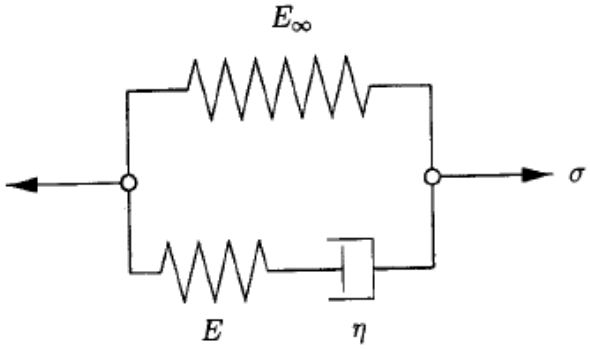

Figure-1: Zener rheological model

In the rheological model, (figure 1), E is the spring constant of the Maxwell branch, η is its dashpot viscosity coefficient and E_∞ is the constant of the spring related to the equilibrium state ε and σ are the total deformation and the stress respectively. The deformation of the dashpot of the Maxwell branch is denoted α and the free energy density is denoted Ψ. Note that this model is governed by the following equations:

$$\sigma = E_\infty \varepsilon + E\left(\varepsilon - \alpha\right) \tag{1}$$

$$\eta\dot{\alpha} = E\left(\varepsilon - \alpha\right) \tag{2}$$

$$\Psi = \frac{1}{2}E_\infty \varepsilon^2 + \frac{1}{2E}\left(E\varepsilon - Q\right)^2 \tag{3}$$

$$\dot{Q} + \frac{1}{\tau}Q = \frac{\gamma}{\tau}\frac{\partial \psi_o}{\partial \varepsilon} \tag{4}$$

In Equations (2) and (4) the dot (.) denotes the derivative of this variable with respect to time. Note that $\gamma = E/E_o$ is a positive constant, E_o is the instantaneous modulus of the material model and $\tau = \eta/E$ is the relaxation time constant. The instantaneous potential mentioned in equation (4) is the instantaneous elastic energy defined as:

$$\psi_o = \frac{1}{2}E_o\varepsilon^2 \tag{5}$$

Considering equations (1), (2), (3), (4) and (5), one can obtain the expression of the Cauchy stress:

$$\sigma = \int_0^t \left(\left(1-\gamma\right) + \gamma \exp\left(-\frac{t-t'}{\tau} \right) \right) \frac{\partial \psi_o}{\partial \varepsilon} dt' \tag{6}$$

2.2 Nonlinear viscoelastic model with reduced time

The concept of reduced time was introduced essentially to describe thermorheologically simple materials' behavior. For such materials, their behaviors are affected by the history of temperature via a single scalar-valued function called temperature shift function [8] [9]. This concept was generalized to investigate thermorheologically complex materials' behavior by a temperature and stress/ strain dependant shift function (Schapery [10], Matsuoka [11] and McKenna [12]).

In what follows a viscoelastic model with reduced time is proposed.

The free energy density proposed by Simo [3] is considered. Remember that the author postulated a free energy as a Taylor series of the right Cauchy-Green strain tensor and an internal variable $\underline{\underline{Q}}$: overstress or non-equilibrium stress (theory of thermodynamic internal variables). Simo [3] proposed an evolution law of this variable. Our approach consists on resolving this law in order to obtain the expression of the internal variable as an integral of the history of deformation/ stress, and then we replace the variable $\underline{\underline{Q}}$ by its expression in the energy. So, we obtain a free energy density in a functional form, then we follow the approach of Christensen [1] to derive constitutive equations.

The evolution law of the internal variable, the free energy and the reduced time function are postulated as follow:

$$\frac{d\underline{\underline{Q}}}{d\xi}+\frac{1}{\tau}\underline{\underline{Q}}=\frac{\gamma}{\tau}\left[2\frac{\partial\psi_o\left(\underline{\underline{C}}\right)}{\partial\underline{\underline{C}}}\right] \tag{7}$$

$$\xi(t)=\int_0^t\frac{dt'}{a\left(\underline{\underline{C}}\right)} \tag{8}$$

$$\Psi=\psi_o\left(\underline{\underline{C}}\right)-\frac{1}{2}\underline{\underline{Q}}:\underline{\underline{C}}+\frac{1}{4\mu_o\gamma}\underline{\underline{Q}}:\underline{\underline{Q}} \tag{9}$$

In equation (9) μ_o denotes the initial shear modulus of the material. Considering these equations and using the methodology, described above, one can obtain the constitutive equation of the second Piola-kirchhoff stress tensor:

$$\underline{\underline{S}}=\int_0^{\xi}G\left(\xi-\xi'\right)\frac{\partial}{\partial\xi'}\left(2\frac{\partial\psi_o\left(\underline{\underline{C}}\right)}{\partial\underline{\underline{C}}}\right)d\xi' \tag{10}$$

$G(t)$ is a time dependant exponential decay function, known as the relaxation function. The equation of this material characteristic function can be expressed as:

$$G(t)=\gamma_\infty+\sum_i\gamma_i\exp\left(-\frac{t}{\tau_i}\right) \tag{11}$$

3 IDENTIFICATION

The aim of this section is to present the identification procedure of the hyperelastic potential, the relaxation function and the reduced time function.

3.1 Identification of hyperelastic potential and relaxation function

The identification of the hyperelastic potential $\psi_o\left(\underline{\underline{C}}\right)$ and the relaxation function $G(t)$ equation (11) is provided using ABAQUS software and simple extension tests at equilibrium state; i.e. low strain rates and relaxation at different levels of deformation. In figure 2 these two functions are plotted using experimental data and the one obtained by the identification. Note that the used hyperelastic potential is a third order Ogden law [13]. For such material the hyperelastic potential is expressed as follows:

$$W = \sum_{n=1}^{3} \frac{\mu_n}{\alpha_n} (\lambda_1^{\alpha_n} + \lambda_2^{\alpha_n} + \lambda_3^{\alpha_n} - 3) \tag{12}$$

$\lambda_i \; i = 1..3$ are the principle stretches of the Cauchy-Green strain tensor and $\alpha_n \; and \; \mu_n \; ; n = 1..3$ are material's constants to be identified.

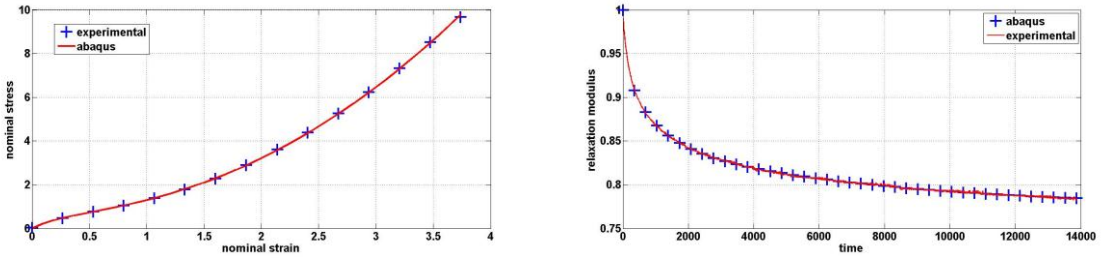

Figure 2: Hyperelastic behavior and relaxation curves

3.2 Identification of reduced time function

In order to satisfy thermodynamic principles via Clausius-Duhem inequality the deformation shift function $a\left(\underline{\underline{C}}\right)$ should be a nonnegative function of the right Cauchy-Green strain tensor.

Two identification approaches was investigated to determine this function. The first one consists on postulating that this function as a Mooney-Rivlin potential (Equation (13)). Several parameters are then identified using a minimization procedure on the error between experimental and theoretical stress values.

Whereas, the second one consists on expressing the error between theoretical and experimental second Piola-Kirchhoff stresses for pure shear test (15) and simple extension test (14) as a function of the reduced time $\xi(t_i)$ at each experimental time t_i. For a given experimental stress the corresponding reduced time is obtained thanks to a minimization procedure over the absolute error function (equations (14) and (15)) using MATLAB software. Numerical values of the reduced time are obtained for each experimental time. A least square fitting procedure is then used to determine the form of the reduced time function.

$$a\left(\underline{\underline{C}}\right) = c_1\left(I_1 - 3\right) + c_2\left(I_2 - 3\right) \tag{13}$$

$$E_{rr}^{a}\left(\xi\right) = \left| \sum_{i=1}^{2} \frac{2\mu_i}{\alpha_i} \left(\lambda(\xi)^{-\alpha_i} + \lambda(\xi)^{-\frac{1}{2}\alpha_i - 1} \right) + G_0 \int_0^{\xi} \sum_{i=1}^{2} \left(\frac{g_i}{\tau_i} \exp\left(-\frac{\xi - \xi'}{\tau_i} \right) \right) \times \sum_{i=1}^{2} \frac{2\mu_i}{\alpha_i} \left(\lambda(\xi')^{-\alpha_i} + \lambda(\xi')^{-\frac{1}{2}\alpha_i - 1} \right) d\xi' - S^{\exp} \right| \tag{14}$$

$$E_{rr}^{a}(\xi)=\left|\begin{array}{l}\displaystyle\sum_{i=1}^{2}\frac{2\mu_i}{\alpha_i}\left(\lambda(\xi)^{-\alpha_i}+\lambda(\xi)^{-\alpha_i-1}\right)+\\ \qquad\displaystyle G_0\int_0^{\xi}\sum_{i=1}^{2}\left(\frac{g_i}{\tau_i}\exp\left(-\frac{\xi-\xi'}{\tau_i}\right)\right)\times\sum_{i=1}^{2}\frac{2\mu_i}{\alpha_i}\left(\lambda(\xi')^{-\alpha_i}+\lambda(\xi')^{-\alpha_i-1}\right)d\xi'-S^{\exp}\end{array}\right| \quad (15)$$

4 CONCLUDING REMARKS

In this paper, a nonlinear viscoelastic model for rubber-like materials is presented. It's able to predict the elastomers behavior in the whole range of deformation and for several strain rates. As a first step, a rheological motivation was investigated to build a one dimensional viscoelastic model. Then, a three dimensional model was developed thanks to a combination between two approaches: the functional approach and the internal variable approach. The dependence of the material's properties on the strain was illustrated using a reduced time instead of the real time which is a function of the deformation history.

The identification procedure of several parameters of the model was studied. For the identification of relaxation modulus/hyperelastic potential and reduced time function, experimental data of pure shear and uniaxial extension tests, on a natural rubber NR, was used with a combination of ABAQUS and MATLAB software respectively.

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