EXTENSION OF THE VARIATIONAL THEORY OF COMPLEX RAYS TO ORTHOTROPIC SHELL STRUCTURES

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ABSTRACT

Recently the interest of aerospace and automotive industries toward the study of the vibrational response of orthotropic shell structures has grown rapidly. The low and high-frequency responses can be correctly simulated by the Finite Element Method (FEM) or the Statistical Energy Analysis (SEA) respectively. Over the last few years some Trefftz methods such as the Variational Theory of Complex Rays (VTCR) has been proposed to address the medium-frequency range. In this paper the extension of the VTCR to orthotropic shell structures has been developed. The theory has been generalized to orthotropic materials and a significant numerical example has been proposed to illustrate the effectiveness of the method.

1 INTRODUCTION

The increased use of composite shell structures has fostered interest towards virtual testing of vibrational behavior of orthotropic shell structures. In literature there are many methods optimized to investigate a vibrational problem is a specific frequency range. [1] reports a detailed description of these approaches. The present work focus on the mid-frequency range extending the applicability of the Variational Theory of Complex Rays (VTCR) [2] to orthotropic shell structures. This method approximates the vibrational problem solution as a sum of shape functions that identically satisfy equilibrium equations and addresses boundary conditions in weak form. This approach allows *a priori* independent approximations among subdomains granting flexibility and robustness. VTCR has been already implemented in shallow shell theory [3] and for orthotropic plates [4].

The present work extends the VTCR to orthotropic shell structures. First, the general shell-VTCR theory is presented and corrections are introduced for orthotropic shells. After that, a relevant numerical example is investigated to validate the strategy.

2 SHELL - VTCR

We will refer to the notation introduced in [5] where the general shell theory is described. Since the VTCR is a Trefftz method, the solution is searched in a function set that satisfy equilibrium equations. Boundary and corner residuals are addressed in weak form $\mathbf{B} = \mathbf{l}$ where $\mathbf{B}$ is the bilinear form, $\mathbf{l}$ is the linear form being VTCR a Galerkin method. [3] reports a more detailed version of the weak variational formulation.

Since VTCR is a Trefftz method, any kind of shape function f_{SFi}, proved that satisfy equilibrium equations, can be chosen as solution in subdomain Ω_i. In the present work plane waves are used

$$f_{SF}(\mathbf{x}_{reli}) \approx \sum_{l=1}^{n} a_{li}\hat{\mathbf{c}}_{li} e^{j\mathbf{k}_{li}\mathbf{x}_{rel}}, \tag{1}$$

where a_{qi} are amplitude coefficients determined by the weak form, $\mathbf{k}_{li}$ is the wave vector, $\hat{\mathbf{c}}_{li}$ unit direction vector, and $\mathbf{x}_{reli}$ is the relative position vector in curvilinear coordinates $\{\alpha_i, \beta_i\}$. Without loss of generality, the wave vector can be divided in the wavenumber k_{li} and the unit direction vector of the wave vector $\hat{\mathbf{k}}_{li}$

$$\mathbf{k}_i(l) = \mathbf{k}_{li} = k_{li}\hat{\mathbf{k}}_{li}. \tag{2}$$

k_{li} and $\hat{\mathbf{c}}_{li}$ are chosen so that equilibrium equations are identically satisfied. The discretization is performed on $\hat{\mathbf{k}}_{li}$. Two kind of plane waves are needed: evanescent and propagative. The difference lies on $\hat{\mathbf{k}}_{li}$. It is

$$\hat{\mathbf{k}}_{li} = \mathbf{L}_i\mathbf{O}_i\mathbf{T}_{li} \cdot \mathbf{p} \tag{3}$$

$$\mathbf{T}_{li} = \begin{bmatrix} \cos(\theta_{li}) & -\sin(\theta_{li}) \\ \sin(\theta_{li}) & \cos(\theta_{li}) \end{bmatrix}, \quad \mathbf{O}_i = \sqrt[8]{D_{\alpha i}D_{\beta i}} \begin{bmatrix} D_{\alpha i}^{-1/4} & 0 \\ 0 & D_{\beta i}^{-1/4} \end{bmatrix}, \quad \mathbf{L}_i = \begin{bmatrix} L_{\alpha i} & 0 \\ 0 & L_{\beta i} \end{bmatrix} \tag{4}$$

where $\mathbf{p} = [1, 0]'$ for propagative waves and $\mathbf{p} = [\cosh(\phi_{mi}), j\sinh(\phi_{mi})]'$ for evanescent waves, $L_{\alpha i}$ and $L_{\beta i}$ are Lamé parameters, θ_{li} is the discretization angle over the unit circle, and ϕ_{mi} is a real parameter that controls between the oscillatory and the evanescent part of the evanescent wave. Figure 1 reports their qualitative behavior. $\mathbf{L}_i$ and $\mathbf{O}_i$ are correction matrices for orthotropic materials.

(a) Propagative waves. (b) Evanescent waves.

Figure 1: Qualitative behavior of the propagative and evanescent waves described in Section 2.

3 NUMERICAL RESULTS

Figure 2 illustrates geometry of a complex frame structure and the amplitude magnitude of the VTCR solution. Three sub-domains are connected by the same edge. The first two are cylinder parts while the last one is a plate. All boundaries are clamped but left edge where an out-of-plane oscillatory distributed load $\mathbf{p} = [1, 0, 0]' e^{i\omega t}$N/m is applied. For the sake of simplicity thicknesses are constant $h_1 = h_2 = h_3 = 3$mm as well as the damping factor $\eta = 0.001$. Table 1 reports material properties as well as frequency.

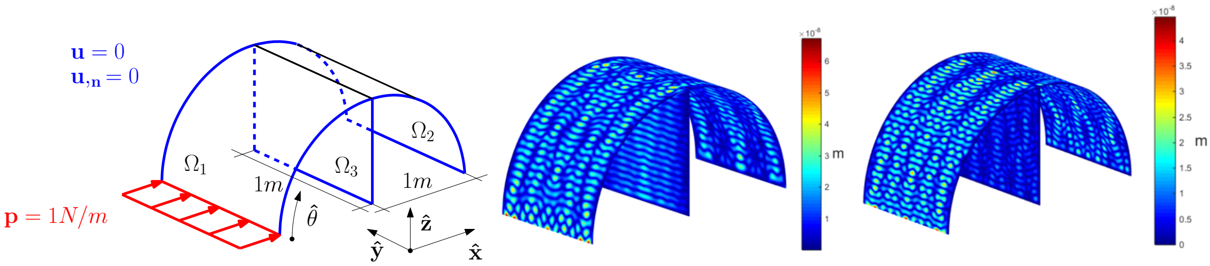
f	3700	Hz
$E_{\theta 1} = E_{\theta 2} = E_{z3}$	125	GPa
$E_{y1} = E_{y2} = E_{y3}$	60	GPa
$G_{\theta y1} = G_{\theta y2} = G_{zy3} i$	18	GPa
$\nu_{\theta y1} = \nu_{\theta y2} = \nu_{zy3}$	0.3	
$\rho_1 = \rho_2 = \rho_3$	2000	Kg/m^3

Table 1: Orthotropic material properties and frequency examined of the numerical example described in Section 3.

The VTCR implemented in MATLAB® is compared with a FEM reference generated by ABAQUS® . The two programs are run on the same workstation and performances compared. The error based on kinetic energy is

$$err = \frac{|E_K(\mathbf{u}_{FEM}) - E_K(\mathbf{u}_{VTCR})|}{E_K(\mathbf{u}_{FEM})}. \tag{5}$$

In this case the error is $\approx 8\%$ due to small theory differences. Computational costs are illustrated in Table 2. FEM mesh must be very refined to counteract the pollution effect [6]. For this reason, VTCR greatly outperforms FEM in terms of time and memory consumption.



(a) Geometry. (b) VTCR. (c) FEM.

Figure 2: Geometry, VTCR and FEM solutions of the frame structure described in Section 3..

	Time consumption	Memory consumption
FEM	1153 [s]	10 [Gb]
VTCR	4 [s]	70 [Kb]

Table 2: Performances comparison of the numerical example described in Section 3

4 CONCLUSIONS

Corrections for orthotropic materials were introduced in the general shell-VTCR theory. Since at mid-frequency FEM suffers of pollution error, FEM mesh must be very refined. For this reason, VTCR greatly outperforms FEM at mid-frequency.

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