APPLICATION OF NON-PARAMETRIC UNCERTAINTIES METHOD ON LAUNCHERS MECHANICAL SPECIFICATIONS

S. Muller[1], A. Alouani[13]

[1]AIRBUS Defence and Space, Department Mechanical Engineering
66 route de Verneuil, F-78133, Les Mureaux Cedex, France
Email: stephane.muller@astrium.eads.net, antoine.alouani2@astrium.eads.net

ABSTRACT

The launch vehicles are subject to severe dynamic loads at lift-off and during flight ascent. Moreover, a major part of European launch vehicles are of composite construction. Thus, a robust design requires a proper consideration of uncertainties in excitations and materials. A non-parametric methodology was experienced on the condensed finite element models of parts of the ARIANE 5 launcher with the objective of releasing less dimensioning but still justified mechanical specifications to get used by the launcher sub-contractors. Such methodology allows introducing different level of uncertainties on parts of the launcher depending on the complexity of elements and their impact on the dynamic phenomenon targeted. The article details the methodology implementation already achieved on ARIANE 5 on the solid rocket booster pressure oscillation load case which is one of the driving load case regarding the amplitude of the low frequency vibrations on the launcher.

1 INTRODUCTION

A launcher FEM is the result of the assembly of numerous sub-structures FEM provided by sub-contractors. Uncertainties at launcher level cover not only scattering of materials and modeling but include also discrepancies introduced by operational conditions of use (with different boundaries conditions than the ones used to set-up and validate the sub-structure FEM) and by connections modeling between sub-structures. Mastering of uncertainties in structural dynamics is hence a challenge that can be handled through a various set of methodologies. [1] gives a global overview of the research field that can be subdivided, on an engineering perspective, into "microscale" schemes dealing with physical properties of FE elements and "macroscale" schemes dealing with mass, stiffness, damping matrices properties of a FE sub-component.

- Microscale (parametric) methods are adapted to small models but require prohibitive CPU times when applied to large FE models like in a launcher modeling case. Furthermore, they take only into account physical parameter uncertainties.
- Macroscale (non-parametric) methods are introducing uncertainties at a macroscopic level of analysis, e.g. on matrices of super-elements of the assembled FEM. The main methods include Gaussian orthogonal ensemble, non-parametric approaches and matrix scaling.

These approaches, coupled with simulation methods (Monte-Carlo, Factorial Design) are much less CPU demanding and allow introducing macroscopic perturbation covering more than physical properties uncertainties. The main drawback is that the link between tuning factors and design parameters is not as straightforward as for the local methods. The interpretation of the level of uncertainties and its physical likelihood is less direct. A first trade-off between the methods led us introducing uncertainties in the launcher FEM through global approaches, e.g. matrix scaling and non-parametric techniques. Aside CPU time considerations, these methods are also well adapted to the launcher dynamic modeling based on assembly of different Craig-Bampton condensed FEM from sub-contractors that can only be scattered through their mass, stiffness and damping matrices, native FEM being rarely accessible at launcher system level.

1.1 Matrix scaling

The matrix scaling consists in introducing perturbations into mass, damping and stiffness condensed matrices $[\mathrm{M}],[\mathrm{K}],[\mathrm{C}]$ used in the structural dynamics equation (1) with scalar operators $\delta_{\mathrm{M}}, \delta_{\mathrm{C}}, \delta_{\mathrm{K}}$ associated to a probability distribution. Mechanical uncertainties are then characterized by random matrices $[\tilde{\mathrm{M}}],[\tilde{\mathrm{C}}],[\tilde{\mathrm{K}}]$ (2).

$$[\tilde{M}]\{\ddot{X}\}+[\tilde{C}]\{\dot{X}\}+[\tilde{K}]\{X\}=\{F_{ext}\} \tag{1}$$

$$[\tilde{\mathrm{M}}]=\delta_{\mathrm{M}}.[\mathrm{M}],\ [\tilde{\mathrm{C}}]=\delta_{\mathrm{C}}.[\mathrm{C}],[\tilde{\mathrm{K}}]=\delta_{\mathrm{K}}.[\mathrm{K}] \tag{2}$$

We applied this technique for simple sub-structures presenting uniform characteristics (isotropy of material and/or geometry) where the risk of non-physical likelihood introduced by the scaling is limited.

Figure 1: ARIANE 5 payload structure

In the ARIANE 5 launcher case, elements like payload adaptors (Figure 1) are relevant for such a technique. As a result, the matrix scaling is also suited for models condensed statically via the Guyan method.

1.2 Random matrices

This technique [3] is a generalization of the matrix scaling applied on FEM super-elements with stochastic matrices applied on mass, damping and stiffness matrices (3) instead of scalar operator. The random matrices are then defined by:

$$\left[\tilde{\mathrm{M}}\right] = [\mathrm{L}_\mathrm{M}]^\mathrm{T}.[\mathrm{G}_\mathrm{M}].[\mathrm{L}_\mathrm{M}],\ \left[\tilde{\mathrm{C}}\right] = [\mathrm{L}_\mathrm{C}]^\mathrm{T}.[\mathrm{G}_\mathrm{C}].[\mathrm{L}_\mathrm{C}],\ \left[\tilde{\mathrm{K}}\right] = [\mathrm{L}_\mathrm{K}]^\mathrm{T}.[\mathrm{G}_\mathrm{K}].[\mathrm{L}_\mathrm{K}] \tag{3}$$

Where $\left[\mathrm{G}_{M,C,K}\right]$ are stochastic initiation matrices and $\left[\mathrm{L}_{M,C,K}\right]$ are the mass, damping and stiffness matrices expressed by the Cholesky factorization (4).

$$[\mathrm{X}] = [\mathrm{L}_\mathrm{X}]^\mathrm{T}.[\mathrm{L}_\mathrm{X}] \text{ with } \mathrm{X} = \mathrm{M},\ \mathrm{C},\ \text{or } \mathrm{K} \tag{4}$$

Guaranteeing the random matrices being physically admissible, meaning that they give admissible solutions of (1), requires the stochastic initiation matrix $[\mathrm{G}]$ verifying the following conditions [3], called the available objective information:

- Random matrices $[\mathrm{G}]$ are defined in the probability space $(\mathcal{A}, \mathcal{T}, \mathcal{P})$ with values in $\mathbb{M}_n^+(\mathbb{R})$
- The mean values of these random matrices must be equal to $[I]$ so that $\varepsilon\left(\left[\tilde{M}, \tilde{K}, \tilde{C}\right]\right) = [M, K, C]$
- $\varepsilon\left(\left\|[G]^{-1}\right\|_F^2\right) < \infty$, where $\|\ \|_F$ is the Frobenious norm in order to guarantee that the matrix inverse always exists,

Nevertheless, the amount of uncertainty introduced in the model via this technique can still be assessed thanks to a scalar quantity. This so called uncertainty tuning parameter δ applied on $[\mathrm{M}]$, $[\mathrm{C}]$ or $[\mathrm{K}]$ is defined by (5):

$$\delta = \sqrt{\frac{\left\|[\mathrm{G}] - [\mathrm{I}]\right\|_\mathrm{F}^2}{\left\|[\mathrm{I}]\right\|_\mathrm{F}^2}} \tag{5}$$

As a result, an infinite amount of stochastic matrices [G] corresponding to the same level of global uncertainty can be generated. This parameter δ would thus be an equivalent of the standard deviation of a scalar uncertainty: infinity of random values can be generated for a given standard deviation in accordance with a defined distribution law. The distribution law associated to those matrices [3] has been defined in order to respect the available objective information and to minimize the entropy introduced in the system:

$$\mathrm{P}_{[\mathrm{G}]}([\mathrm{G}]) = 1_{\mathrm{M}^+(\mathrm{R})}([\mathrm{G}]) \cdot \mathrm{C}_\mathrm{G} \cdot \det([\mathrm{G}])^{(\mathrm{n}+1)\frac{(1-\delta^2)}{2\delta^2}} \cdot \mathrm{e}^{\frac{-\mathrm{n}+1}{2\delta^2} tr([G])} \tag{6}$$

Where:

- $\det([\mathrm{G}])$ is the determinant of the [G] matrix,
- $tr([G])$ its trace, n its dimension,
- $1_{\mathrm{M}^+(\mathrm{R})}$ is a function equal to 1 as the matrix belongs to $\mathrm{M}^+(\mathrm{R})$, and zero otherwise,
- C_G is a positive normative constant, detailed in [3].

Unlike the matrix scaling method where the uncertainties are introduced uniformly on the FE matrices of each super-element, the non-parametric methodology allows introducing local and independent uncertainty factors.

2 A STUDY CASE –ARIANE5 SOLID ROCKET BOOSTERS FIRST ACOUSTIC MODE LOAD CASE

2.1 Load case characteristics

One of the main mechanical load case encountered in ARIANE 5 flight is the first Solid Rocket Boosters (SRBs) acoustic mode load case [2]. This load case is characterized by sine bursts excitations generated by both SRBs (Figure 2) and gives signification vibration responses on all parts of the launcher.

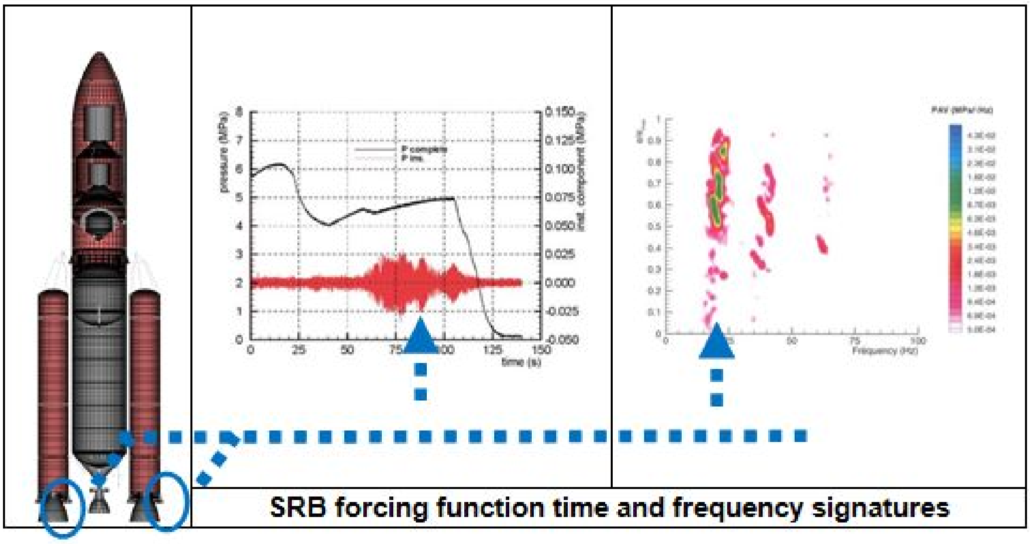

Figure 2: ARIANE 5 SRBs first acoustic mode load case

The flight analyses show that the vibrations levels are generated mostly by global modes of the launcher (bending modes, tank modes) that are dynamically driven by global mechanical characteristics of launcher sub-components. It is then well suited for the non-parametric uncertainties methodology applied on mechanical consistent ensembles (stages, skirt…). In consequence, we cut the launcher FEM into super-elements (Figure 3) on which uncertainties could be applied.

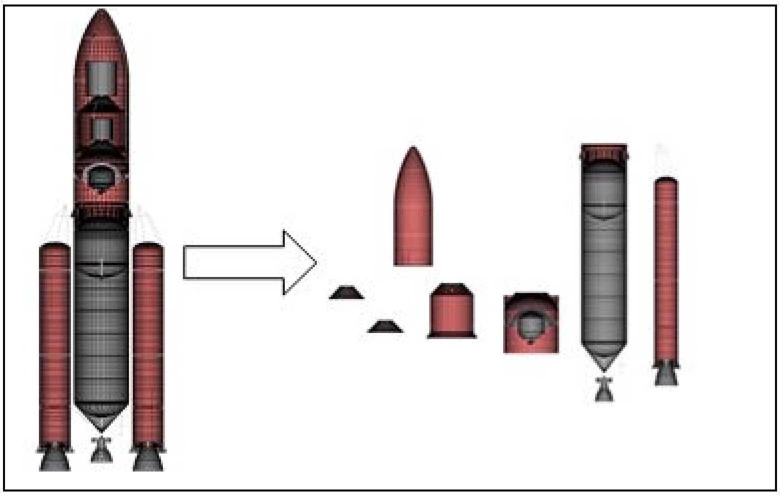

Figure 3: Launcher cutting into elementary mechanical models

This includes Craig-Bampton (non-parametric uncertainties) and Guyan (matrix scaling) condensation schemes as well as explicit modeling (matrix scaling) for some simple elements. As the relevant values of uncertainty tuning parameters δ on the different launcher parts cannot be fixed thanks to physical considerations, an inverse problem has to be solved. The aim is then to find values for these parameters able to predict responses consistent with what has been observed in flight and also with a certain degree of conservatism in order to have a robust simulation tool. The identification of uncertainties factor to get applied on super-elements to reach a relevant coverage of predictions requires metrics in both frequency and time domain. ARIANE 5 load case predictions are indeed performed in these two spaces with a global objective of releasing 99% envelope vibration levels.

2.2 Time domain (A_t) & frequency domain (A_f) sensitivity metrics

In the time domain, the metric is defined as the scalar ratio between maximum values of 99% envelope and nominal prediction (7).

$$A_t = \frac{\max_t(\gamma_{99\%}(t))}{\max_t(\gamma_{nominal}(t))} \tag{7}$$

In the frequency domain, the metric is defined by the weighted sum of the mean amplification ratios at resonance calculated on the different peaks computed through shock spectra responses (8).

$$A_f = \sum_{i=1}^{N_{peak}} C_i \cdot \frac{\int_{f_1^{nom}(i)}^{f_2^{nom}(i)} \gamma_{99\%}(f)df}{\int_{f_1^{nom}(i)}^{f_2^{nom}(i)} \gamma_{nominal}(f)df} \quad \text{with } C_i = \frac{\max\limits_{f\in[f_1^{nom}(i),f_2^{nom}(i)]}(\gamma_{99\%}(f))}{\sum_i^{N_{peak}} \max\limits_{f\in[f_1^{nom}(i),f_2^{nom}(i)]}(\gamma_{99\%}(f))} \tag{8}$$

The 99% envelopes are assessed through quantiles computations based on the several thousands of simulations representing one single uncertainty case where different super-elements are scattered with a specific value of uncertainty parameter δ (Figure 4).

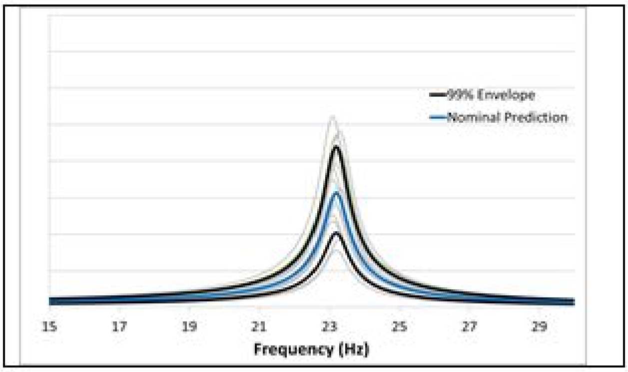

Figure 4: Nominal and 99% predictions

2.3 Calibration of δ_M and δ_K

As a first step, unitary sensitivity studies were performed to identify driving uncertainty factors w.r.t dynamic responses of the launcher. Each super-element was scattered separately considering four values of tuning factor δ_M and δ_K selected between 10% and 40%, represented each time by about 2000 draws of different stochastic matrices [G] introducing the required global degree of uncertainty on the related launcher part. Each assembled model was then used for a vibration prediction corresponding to the SRBs first acoustic mode load case in both frequency and time domains Using coverage metrics A_t and A_f, sensibility plots were computed on every launcher's point of interest corresponding to sensor locations in order to compare the results with the flight data. Families were identified based on their visual similarity of behavior and their spatial location on the launcher. Similar footprints were identified on spatial location families on the launcher which allowed defining spatially correlated zone regarding the load case dynamic responses on spatial zones. An example is given for the blue delimited zone on the Figure 5.

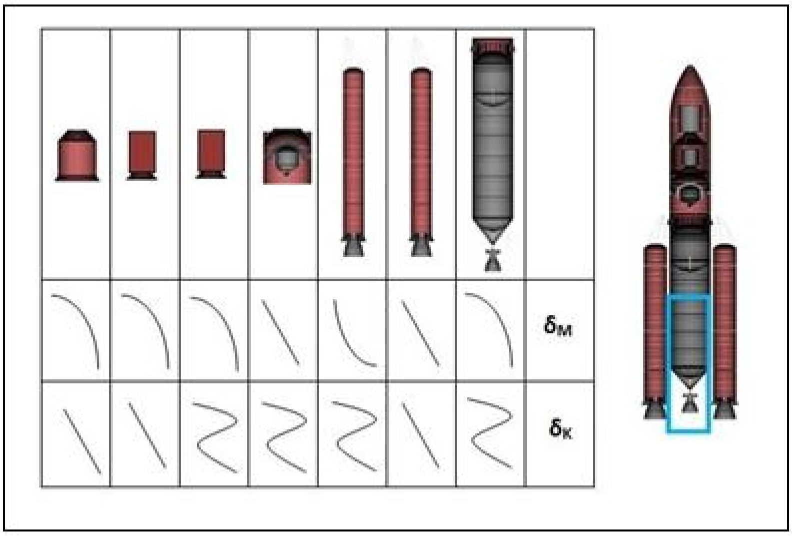

Figure 5: Sensibility footprints vs. mechanical models within a spatial family

The family segmentation gave similar results when performed in time and frequency domains, which looks logical as the physic of the load case is unique behind the time or frequency approaches. Hence, the non-parametric methodology allows putting uncertainties on a FEM on a limited spatial zone. This opens a large spectrum of possible tuning consistent with uncertainties of one or several super-elements. Nevertheless, the single footprints, if useful to have a first trend of the relative importance of uncertainties applied on super-elements, are not adapted to multi-variable uncertainties tuning; this requires setting-up multi-dimensional surfaces whose exhaustive computation would be still too demanding regarding CPU time.To overcome this drawback, an optimized factorial design coupled with relevant simplifications was set-up in order to estimate reliably the response surfaces approximating the responses amplification as a function of the various uncertainties within an acceptable amount of computations.

3 SURFACE RESPONSES FACTORIAL DESIGN APPROACH

3.1 Modeling of factorial design

The aim of the factorial design is to have an analytical formulation as accurate as possible in order to predict quickly the launcher responses for any set of uncertainties applied on its structures without generating the 2000 stochastic matrices or computing the corresponding responses with the FEM. The surface response was addressed through a Taylor development of the computed sensitivity metrics (9).

$$A_{t/f}(X_i, X_j, X_k, X_l, X_m) = \alpha_0 + \sum_i \alpha_i X_i + \sum_{(i,j),i\le j} \alpha_{ij} X_i X_j + \ldots + \sum_{(i,j,k,l,m),i\le j\le k\le l\le m} \alpha_{ijklm} X_i X_j X_k X_l X_m + \varepsilon \quad (9)$$

With:

- $A_{t/f}$: the time/frequency coverage of the response in one point of the scattered launcher with respect to the nominal response (no uncertainty in the model)
- X_i : centered uncertainty level (mass/stiffness) applied to the super-element i, $X_i = \frac{\delta_i - 0.2}{0.2}$
- α_i : response surface coefficients to be identified
- ε : residue

The $N = 14$ dimension corresponds to the number of super-elements considered for the non-parametric method, e.g. stiffness and mass uncertainties tuning factors applied on 7 super-elements. The polynomial form is in accordance with sensibility footprints that didn't put in evidence any steep variations but rather continuous evolutions. The factorial design setting-up is based on the selection of optimal sets of values for input parameters $(X_i, X_j, X_k, X_l, X_m)$ allowing the minimization of the residual factors $\varepsilon\{\varepsilon_i\}$ for P realization with $P \leq N$. The identification problem of the response surface coefficients can thus be defined by the linear system (10).

$$\begin{Bmatrix} y_1 \\ \vdots \\ y_N \end{Bmatrix} = [X] \times \begin{Bmatrix} \alpha_i \\ \vdots \\ \alpha_P \end{Bmatrix} + \begin{Bmatrix} \varepsilon_i \\ \vdots \\ \varepsilon_N \end{Bmatrix}, X \in M^{(N,P)}(\Re) \tag{10}$$

With:

- y : realization vector gathering the results of the experiences attempted
- X : factorial design matrix gathering the selected values of the input parameters and defining the different independent experiences performed to have the realization vector.

Such system is classically solved through a regression method by increasing the number of experiences from P to N and computing most likely parameters $\tilde{\alpha}$ (11).

$$\tilde{\alpha} = \left(X^T \cdot X\right)^{-1} \cdot X^T \cdot \{Y\} \tag{11}$$

Nevertheless, with formulation (8), the resolution of the order N exhaustive Taylor development requires to identify a very large number of coefficients (12):

$$\dim = 1 + \sum_{i=1}^{N} C_i^N + N \cdot (N - i) \cdot C_{i-1}^{N-1} \tag{12}$$

For 14 variables, it is equivalent to 761.856 coefficients, so potentially as many experiences to perform, representing up to more than 3000 years of CPU time. As a result, such a problem cannot be solved now, except by massive parallelization of CPU's. Thus, dimension of the problem was reduced (13) in order to ensure admissible computation time but still maintaining an acceptable accuracy of the response surface. A factorial design approach based on Rechtshaffner formulations [5] was set-up accordingly.

$$y = \alpha_0 + \sum_i \alpha_i X_i + \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \alpha_{ij} X_i X_j \tag{13}$$

Relative weights on applied uncertainties can be assessed through $\alpha_{ij,(i,j),i<j}$ (order 2 interaction) and $\alpha_{ij,k,(i,j,k),i<j<k}$ (order 3 interaction) of (9). Their importance is determined by the measurement of the difference in predicted levels with and without this term (Table 1).

$\varepsilon_{ij} = \max\limits_{\delta}\left(\left\lvert\dfrac{\alpha_{ij}\delta^2}{\alpha_0 + (\alpha_i + \alpha_j)\delta + (\alpha_{ii} + \alpha_{jj})\delta^2}\right\rvert\right)$ 2^{nd} order interaction metric
$\varepsilon_{ijk} = \max\limits_{\delta}\left(\left\lvert\dfrac{\alpha_{ijk}\delta^3}{\alpha_0 + (\alpha_i + \alpha_j + \alpha_k)\delta + (\alpha_{ij} + \alpha_{ik} + \alpha_{jk})\delta^2 + (\alpha_{ii} + \alpha_{jj} + \alpha_{kk})\delta^2}\right\rvert\right)$ 3^{rd} order interaction metric

Table 1: Cross-interaction metrics

The Figure 6 presents a typical interaction plot where the ground surface is offset at 15%, meaning that only differences higher than this threshold appear. The stiffness uncertainties cross-interactions are presented, IxJ tag corresponding to the cross interaction between super-elements I and J along the different response locations.

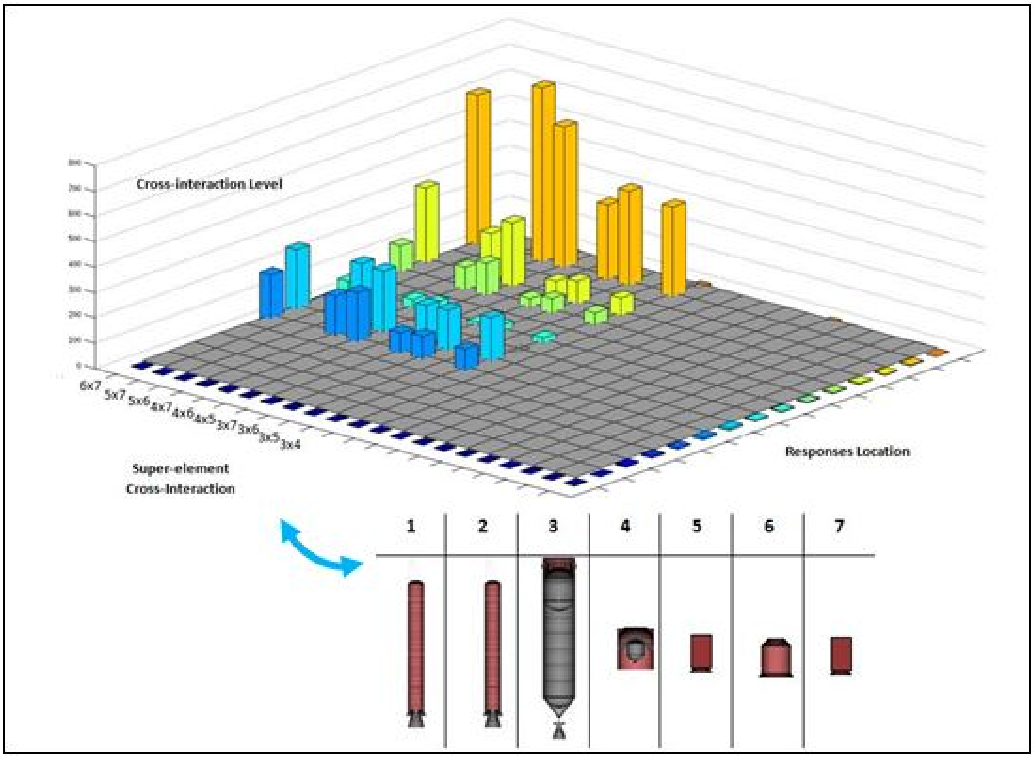

Figure 6: Stiffness cross-interactions between super-elements

The factorial design approach is thus a powerful tool that will be used to set-up a generic uncertainties treatment to be used to release justified and less dimensioning specifications. As a first step, the methodology was used to tune uncertainty factor required to cover flight vibrations by prediction thanks to the polynomial approximation of the launcher responses amplification.

4 CALIBRATION OF UNCERTAINTIES TUNING FACTORS

The weight to be applied on uncertainties tuning factors can be assessed by analysis of flight vibration records using coverage metrics and the analytical behavior of the response amplification of the scattered launcher given by the response surface (13). These populations were then reused to extrapolate tuning factors to get used for releasing 99% envelope vibration levels. Results presented here are a first flight coverage analyses.

4.1 Flight coverage tuning factors

The coverage level is defined (in the frequency domain) by (14).

$$C_F(\delta_j^M, \delta_j^K) = C_F^{nom/vol} \cdot \mathrm{f}_F(\delta_j^M, \delta_j^K) \tag{14}$$

- $C_F^{nom/vol}$ is the ratio of nominal (e.g. without uncertainties) predicted responses vs. flight measured ones (Figure 7),
- f_F is the response surface approximating the response amplification of the scattered launcher with respect to the nominal one via the polynomial expression (13).

The frequency analyses of flight records were reached by shock spectra analysis.

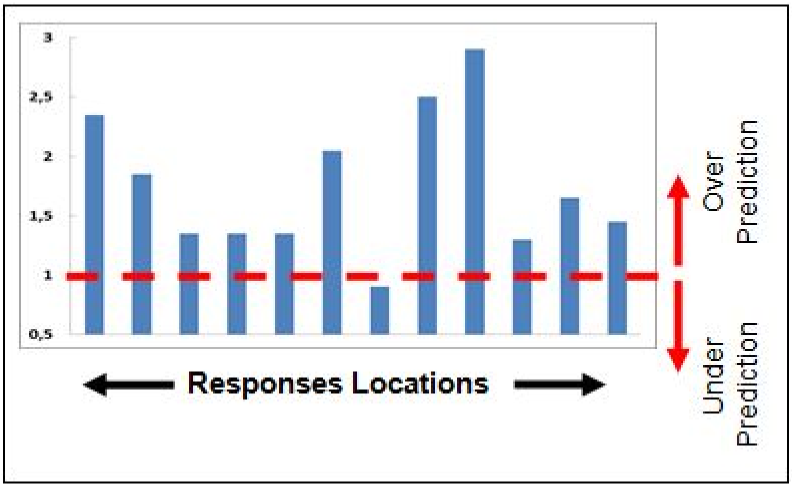

Figure 7: Flight average coverage

A vector objective function (15) was defined on the uncertainty vector $\delta = (\delta^M_1, \ldots, \delta^M_7, \delta^K_1, \ldots, \delta^K_7)$ and optimization algorithms were used to find tuning parameters allowing prediction being as close as possible as flight observations.

$$\overrightarrow{F_{obj}(\delta)} = \left\{ \left| C_F \cdot f_F(\delta) - 1 \right| \right\} \tag{15}$$

The optimizations were realized through Monte-Carlo simulations that demonstrated a convergence of uncertainties tuning parameters into limited intervals.

The table 2 illustrates the results for stiffness uncertainties.

Monte-Carlo populations	Number of solutions	Tuning parameters intervals					
		SE-1&2	SE-3	SE-4	SE-5	SE-6	SE-7
50 k	0	-	-	-	-	-	-
500 k	3	0.3316 0.4129	0.0034 0.0396	0.0168 0.0392	0.0974 0.3594	0.0135 0.0199	0.1459 0.1900
5000 k	81	0.3024 0.4343	0.0002 0.0556	0.0000 0.0621	0.0033 0.4240	0.0003 0.0717	0.1409 0.2027

Table 2: Optimized uncertainties tuning parameters

The solution domain corresponding to a flight event is only a small sub-domain of the 7th dimension space (stiffness uncertainties considered here) but is not singular, meaning that different tuning factors were found admissible regarding the objective function and physical likelihood. Moreover, it has been numerically assessed that the solution sub-domain was continuous: for a specific solution vector δ_0, the vectors δ_0+ $d\delta$ with $|d\delta| << |\delta_o|$ belong also to solutions sub domain. Different convergence algorithms were tested to verify the robustness of the solution domain and find one specific set of uncertainties optimizing the flight coverage by the scatter launcher assembly. They confirmed the tuned domains, without noticeable reduction. These optimums identified thanks to the polynomial approximation of the launcher behavior have been checked by re-computing directly the responses of the corresponding scattered launcher with the finite element model and the related scattered matrices of its sub-structures. Those final direct computations (not the polynomial prediction) for two optimized uncertainties sets are presented in Figure 8 and compared with the nominal simulation by showing the response amplification of 12 observation points with respect to the flight measurements. The unity coverage diagram (plotted in blue) represents the target level of amplification for the optimization process and also the threshold that should not be underpassed.

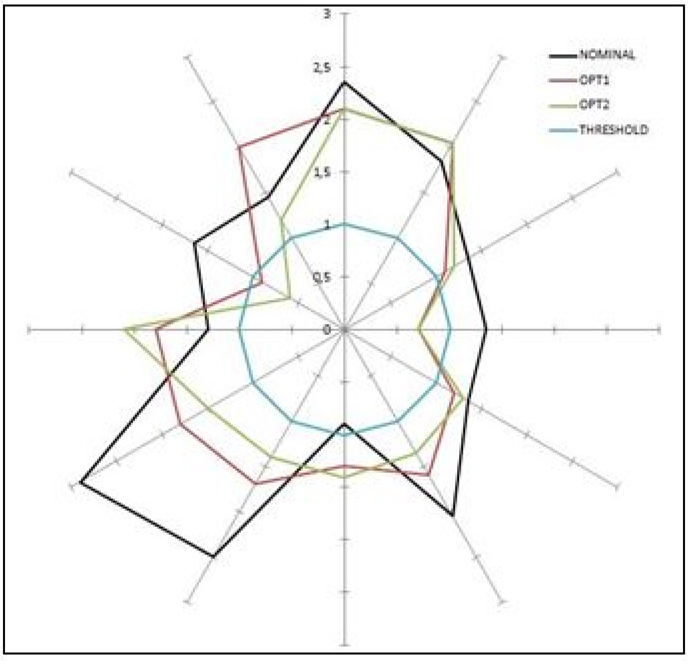

Figure 8: Optimization flight coverage results

The optimal tunings are giving a closer coverage flight than nominal simulation. They are demonstrating that predictions closer to flight can be achieved through application of non-parametric uncertainties technique to super-elements.

5 CONCLUSION

The application of non-parametric uncertainties propagation through factorial design on an ARIANE5 dimensioning load case proved to be fruitful. With a relatively limited time computation, it is possible to analyze unitary and cross-interaction of mass and stiffness characteristics of the main sub-part of the launcher. Using flight measurement feedback allows quantifying and justifying uncertainties set to be applied on the different sub-structures flight by flight.

ACKNOWLEDGEMENTS

This work was performed with the support of CNES R&T Launcher program.

REFERENCES

[1] C.S. Manohar and R.A. Ibrahim. *Progress in Structural Dynamics with Stochastic Parameter Variations: 1987-1998,* Applied Mechanics Review Vol. 52 – Iss. 5, 177-197, 1999

[2] Y. Fabignon, J. Dupays, G. Avalon, F. Vuillot, N. Lupaglazoff, G. Casalis and M. Prévost. *Instabilities and Pressure Oscillations in Solid Rocket Motors,* Aerospace Science and Technology 7 : 191-200, 2003

[3] C. Soize. *Random matrix theory and non-parametric model of random uncertainties in vibration analysis*, Journal of Sound and Vibration, 263 : 893 – 916, 2003.

[4] J. Goupy. *Modélisation par les plans d'expériences*, Techniques de l'Ingénieur, traité Mesures et contrôle, 2000

[5] R. L. Rechtshaffner. *Saturated fraction of* 2^{nd} *and* 3^{rd} *factorial design*, Technometrics, Vol. 9 – No. 4, 1967